JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY

SCHOOL OF BIOLOGICAL AND PHYSICAL SCIENCES

UNIVERSITY EXAMINATION FOR THE DEGREE OF BACHELOR OF EDUCATION

3RD YEAR 1ST SEMESTER 2019/2020

MAIN REGULAR

COURSE CODE: SPH 313

COURSE TITLE: MECHANICS

EXAM VENUE: **STREAM: (B.Ed Sc)**

DATE: **EXAM SESSION:**

TIME: 2:00 HRS

INSTRUCTIONS:

1. **Attempt question 1 (compulsory) and ANY other two questions.**
2. **Candidates are advised not to write on the question paper.**
3. **Candidates must hand in their answer booklets to the invigilator while in the examination room.**

Useful constants

gravitational acceleration, $g, = 9.8$ m s^{-2}
velocity of light in free space $= 3.0 \times 10^8$ m/s
mass of an electron $= 9.11 \times 10^{-31}$

Question 1

(a) (i) Define the moment of a force, hence show that

$$\vec{N} = \frac{\mathrm{d}\vec{L}}{\mathrm{d}t}$$

where the symbols have their usual meanings.

(3 Marks)

(ii) With reference to a specific example, explain what you understand by a conservative force field.

(2 Marks)

(iii) A single conservative force $\vec{F} = (4.0y - 10)\hat{\mathbf{j}}$ N, where y is in meters acts on a particle moving along the y-axis. Determine an expression for the kinetic energy as a function of y, if the particle was initially at the origin.

(4 Marks)

(b) Determine the energy required to give an electron a speed of 0.93 c, starting from rest.

(3 Marks)

(c) Two horizontal forces act on a 2.0 kg chopping board that can slide over a frictionless kitchen counter, that lies in an x-y plane. One force is $\vec{F}_1 = (3.0\hat{\mathbf{i}} + 4.0\hat{\mathbf{j}})$ N. Find the acceleration of the board in unit vector notation when the other force is $\vec{F}_2 = (2.0\hat{\mathbf{i}} + 5.0\hat{\mathbf{j}})$ N.

(3 Marks)

(d) Consider a system of N interacting particles with masses m_i, position vectors $\vec{r}_i$, and momenta $\vec{p}_i = m_i\dot{\vec{r}}_i$ $(i = 1, 2, \cdots, N)$ relative to an inertial frame of reference. The particles are also subject to external forces $\vec{F}_i^{(e)}$, and the masses m_i are assumed to be constant. Show that the equation of motion of the center of mass vector is

$$M\ddot{\vec{R}} = \vec{F}^{(e)}$$

(4 Marks)

(e) State the principle of Galilean transformation and explain its limitations under relativistic conditions.

(2 Marks)

(f) An observer on Earth sees a spaceship at an altitude of 4460 km moving towards the Earth with a speed of 0.970 c, where c is the speed of light in free space. Find the distance from the space ship to the Earth as measured by the captain of the spaceship.

(3 Marks)

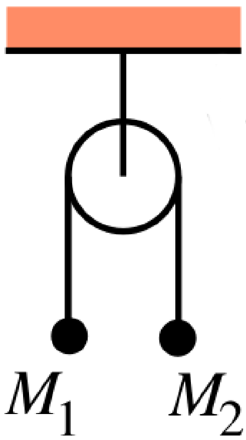

Figure 1: Atwood machine

(g) The two masses in an Atwood machine are m_1 and m_2, where $m_1 > m_2$ (see Fig.1). Determine the acceleration of the system, a, in terms of m_1 and m_2 and state any assumption made.

(4 Marks)

(h) Write down Hamilton's equations for the Hamiltonian

$$H = \frac{1}{2}\left(p^2q^4 + \frac{1}{q^2}\right)$$

(2 Marks)

Question 2

(a) (i) Explain the advantage of using the Lagrangian approach compared to the Newtonian approach in solving problems in mechanics.

(ii) Give the significance of the Hamiltonian.

(1,1 Marks)

(iii) Derive the Hamilton's equations.

(8 Marks)

(b) Use the Hamiltonian method to determine the equation of motion of a simple pendulum placed in a uniform gravitational field

(10 Marks)

Question 3

(a) Two twins are 25.0 years old when one of them sets out on a journey through space at nearly constant speed. The twin in the spaceship measures time with an accurate watch. When he returns to Earth, he claims to be 31.0 years old, while the twin left on Earth knows that she is 43.0 years old. What was the speed of the spaceship?

(4 Marks)

(b) A spacecraft moving at 0.95c travels from the Earth to the star Alpha Centauri, which is 4.5 light years away.

(i) How long will the trip take according to (I) Earth clocks and (II) spacecraft clocks?

(5 Marks)

(ii) How far is it from Earth to the star according to spacecraft occupants?

(2 Marks)

(iii) Determine the speed of the spacecraft as observed by the occupants.

(2 Marks)

(b) Muons have a mean lifetime of 2.2×10^{-6} s when at rest. They are produced at an altitude of 10 km and travel at $0.995c$ toward the earth. Find

(i) The mean lifetime measured on earth.

(ii) The time taken to reach ground level in the earth frame.

(7 Marks)

Question 4

(a) A block of metal of mass m_1 lies on a horizontal table. It is attached to a mass m_2 by a light string passing over a frictionless pulley at the edge of the table (see Figure 2). The

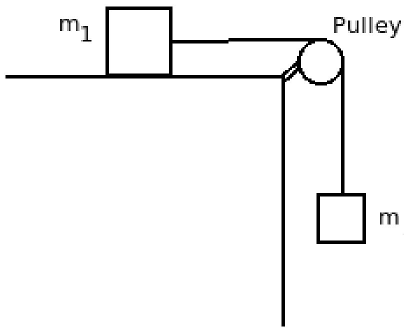

Figure 2:

coefficient of sliding friction between the block and the table is μ.

(i) Determine the acceleration of the system, a, in terms of μ, m_1, m_2 and the gravitational acceleration, g.

(ii) Hence obtain an expression for the tension in the string.

(iii) Given that $m_1 = 1.6$ kg, $m_2 = 0.5$ kg and $\mu = 0.1$, find the acceleration of the system.

(5,2,2 Marks)

(b) Investigate whether the force field defined by

$$\vec{F} = (2y + 8xy^3)\hat{\mathbf{i}} + (2x + 12x^2y^2)\hat{\mathbf{j}}$$

is conservative.

(6 Marks)

(c) A particle of mass 3 kg moves in a force field dependent on time t given by

$$\vec{F} = 24t^2\hat{\mathbf{i}} + (36t - 21)\hat{\mathbf{j}} - 15t\hat{\mathbf{k}}$$

If at $t = 0$ the particle is located at $\vec{r}_0 = 3\hat{\mathbf{i}} - \hat{\mathbf{j}} + 4\hat{\mathbf{k}}$ and has velocity $v_0 = 6\hat{\mathbf{i}} + 15\hat{\mathbf{j}} - 8\hat{\mathbf{k}}$, find the position at any time t.

(5 Marks)

.

Question 5

(a) Explain the meaning of the following terms as used in mechanics:
(i) Generalized coordinates.
(ii) Holonomic constraints.
(iii) Virtual work.

(1,1,1 Marks)

(b) Determine the equations of motion of the masses of an Atwood machine by Lagrangian method.

(6 Marks)

(c) Two masses are connected by a massless rod of length l (see Figure 3). The mass m_1 moves without friction along a horizontal line, while the mass m_2 moves in a plane passing through the line.
(i) Find the Lagrangian of the system.

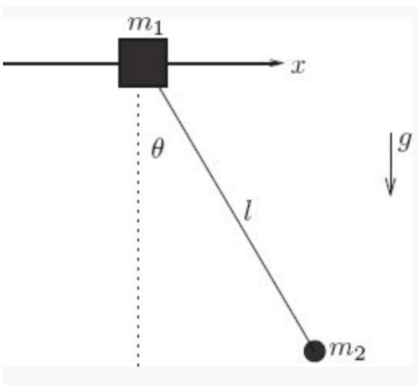

Figure 3:

(ii) Hence find the equations of motion for small oscillations about equilibrium.

(6, 5 Marks)