JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY

SCHOOL OF BIOLOGICAL AND PHYSICAL SCIENCES

UNIVERSITY EXAMINATION FOR DEGREE OF BACHELOR OF EDUCATION (SCIENCE)

4^{TH} YEAR 2^{ND} SEMESTER 2018/2019 ACADEMIC YEAR

MAIN

REGULAR

COURSE CODE: SPH 402

COURSE TITLE: Statistical Mechanics

EXAM VENUE: **STREAM: EDUCATION**

DATE: **EXAM SESSION:**

TIME: 2:00 HRS

Instructions:

1. **Answer question 1 (Compulsory) and ANY other 2 questions.**
2. **Candidates are advised not to write on the question paper.**
3. **Candidates must hand in their answer booklets to the invigilator while in the examination room.**

QUESTION ONE

(a) Using the grand canonical ensemble, evaluate the chemical potential μ(T,P) for an ultrarelativistic gas contained in a box of volume V (6 mks)

(b) Find the entropy S(E,V,N) of an ideal gas of N classical monoatomic particles, with a fixed total energy E, contained in a d-dimensional box of volume V. Deduce the equation of state of this gas , assuming that N is very large (6 mks)

(c) Find the number of ways in which two particles can be distributed in six states if

(i) The particles are distinguishable (2mks)

(ii) The particles are indistinguishable and obey Bose-Einstein statistics (2mks)

(iii) The particles are indistinguishable and only one particle can occupy any one state (2mks)

(d) From observations on the intensities of lines in the optical spectrum of nitrogen in a flame the population of various vibrationally excited molecules relative to the ground state is found as follows :

V	0	1	2	3
N_v/N_0	1.000	0.210	0.043	0.009

Show that the gas is in the thermodynamic equilibrium in the flame and calculate the temperature of the gas (Θ_v = 3350K) (6 mks)

(e) Describe the following concepts as used in statistical mechanics

(i) Microcanonical ensemble (2 mks)

(ii) Canonical ensemble (2 mks)

(iii) Grand canonical ensemble (1mk)

(f) Define the terms microstate and macrostate. Give reasons why we study statistical mechanics (3 mks)

QUESTION TWO

(a) A lattice in a d-dimensional space has N sites; each occupied by an atom whose magnetic moment is μand is in contact with a heat reservoir at fixed temperature T. The atoms do not interact with each other, but they do interact with an applied magnetic field $H = H(r)\hat{z}.$

(i) Express the canonical partition function of this system in terms of a suitable product of integrals over the angle Θ_l between the magnetic moments and the $\hat{z}.$ direction (6 mks)

(ii) In the case when d = 3, find the average magnetization M and the susceptibility per lattice site x (4 mks)

(b) Compute the average energy and the heat capacity of a classical system of N non-identical particles in d spatial dimensions , that has a Hamiltonian of the form

$$H = \sum_{i=1}^{N} A_i |p_i|^s + B_i |q_i|^t$$

The parameter A_i and B_i characterize individual particles while s and t are positive integers and the system is maintained at a fixed temperature T. As a special case, obtain the average energy and heat capacity for N three dimensional harmonic oscillators. (10 mks)

QUESTION THREE

(a) (i) Define density of state (1 mk)

(ii) The density of states functions for electrons in a metal is given by $Z(E)dE = 13.6\times10^{27}E^{1/2}dE$ Calculate the Fermi level at a temperature few degrees above absolute zero for copper which has 8.5×10^{28}electrons per cubic metre (3mks)

(iii) Using the results of problem (i), Calculate the velocity of electrons at the Fermi level in copper (3mks)

(b) For silver (A = 108), the resistivity is $1.5\times10^{3}kg/m^{3}$and Fermi energy $E_F = 5.5eV$. Assuming that each atom contributes one electron for conduction, find the ratio of the mean free path λ to the interatomic spacing d. (7 mks)

(c) Find the probability of occupancy of a state of energy

(i) 0.05 eV above the Fermi energy (2mks)

(ii) 0.05 eV below the Fermi energy (2mks)

(iii) Equal to the Fermi energy. Assume a temperature of 300K (2mks)

QUESTION FOUR

(a) Derive Boltzmann's formula for the probability of atoms in thermal equilibrium occupying a state E at absolute temperature T (15 mks)

(b) If *n* is the number of conduction electrons per unit volume and *m* the electron mass then show that the Fermi energy is given by the expression $E_F = \frac{h^2}{8m}\left(\frac{3n}{x}\right)^{\frac{2}{3}}$ (5 mks)

QUESTION FIVE

a) Calculate the probability that an allowed state is occupied if it lies above the Fermi level by kT, by $5kT$, by $10kT$. (6 mks).

b) When the sun is directly overhead, the thermal energy incident on the earth is 1.4 kWm^{-2}. Assuming that the sun behaves like a perfect blackbody of radius 7×10^5 km, which is 5×10^8 km from the earth show that the total intensity of radiation emitted from the sun is 6.4×10^7 Wm^{-2} and hence estimate the sun's temperature. (6 mks)

c) Estimate the temperature T_E of the earth, assuming that it is in radiation equilibrium with the sun (assume the radius of sun $Rs = 7 \times 10^8$ m, the earth-sun distance $r = 1.5 \times 10^{11}$ m, the temperature of solar surface $Ts = 5,800$ K) (8mks)