JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY

SCHOOL OF MATHEMATICS AND ACTUARIAL SCIENCE

UNIVERSITY EXAMINATION FOR DEGREE OF BACHELOR OF SCIENCE(ACTUARIAL SCIENCE WITH IT)
3RD YEAR 2ND SEMESTER 2019/2020 ACADEMIC YEAR

MAIN CAMPUS

COURSE CODE: SMA 312
COURSE TITLE: OPERATIONS RESEARCH I

EXAM VENUE: **STREAM: BED SCIENCE Y3S2**

DATE:1/12/20 **EXAM SESSION:9-12 NOON**

TIME: 3.00 HOURS

Instructions:

1. **Answer question one (compulsory) and any other two questions.**

1. **Candidates are advised to write on the text editor provided, or to write on a foolscap, scan and upload alongside the question.**
2. **Candidates must ensure that they submit their work by clicking 'FINISH AND SUBMIT ATTEMPT' button at the end.**

QUESTION ONE(30 MARKS)

a) Name four circumstances under which Sensitivity Analysis becomes necessary in Linear Programming (4 marks)

b) State under what circumstances the following inequalities are used in operations research

i) Less or equal to $(\leq)$

ii) Greater or equal to $(\geq)$ (6 marks)

c) Use Gauss Jordan Method to solve the set of simultaneous equations

$$2x_1 + 3x_2 = 14$$
$$x_1 - 2x_3 = 2 \qquad \text{(6 marks)}$$
$$2x_1 + x_2 + 3x_3 = 13$$

b) A firm makes two types of containers, A and B each of which requires cutting, assembly and finishing. The maximum available machine capacity in hours per week for each process is, cutting 50, assembly 84 and finishing 72. The processing time for one unit of each type are as follows

	Time in hours	
Process	A	B
Cutting	4	10
Assembly	8	16
Finishing	8	10

If the profit margin is kshs 600 per unit of A and kshs 1,000 per unit of B.

(i) Formulate a Linear Programming Problem hence state any three suitable methods that can be used in optimization. (7 marks)

(ii) Form a dual of the primal problem in (i) above. (2 marks)

QUESTION TWO (20 MARKS)

a) A firm makes two types of couplings X and Y, each of which requires processing time on lathes, grinders and polishers. The machine time needed for each type of coupling is given in the table below

Coupling type			
	Lathe	Grinder	Polisher
X	6.25	8	5
Y	10	5	2

The total machine time available is 500 hours on lathe, 310 hours on grinders and 160 hours on polishers. The net profit per coupling of type X is shs 90 and of type B is shs 100.

i) Formulate a Linear Programming Problem based on the information above. (4 marks)

ii) Use Graphical Method to advice the firm on how many of each type of coupling it should produce in order to maximize profit. (4 marks)

Due to changing times the profit on coupling X increases to kshs 100 per unit while that of Y deacreses by 40% per unit. Determine whether the optimum level of production in b) ii) above will change and if it does what is the new optimum level of production. (4 marks)

b) Use the Dual simplex method to solve the following LP problem

Minimize $Z = x_1 + 2x_2 + 3x_3$

Subject to

$$2x_1 - x_2 + x_3 \geq 4$$
$$x_1 + x_2 + 2x_3 \leq 8$$
$$x_2 - x_3 \geq 2$$
$$x_1, x_2, x_3 \geq 0$$

(8 marks)

QUESTION THREE(20 MARKS)

a) Use the big M Simplex Technique to solve the Linear Programming problem

Minimise $Z = -2x_1 + 8x_2$

Subject to

$$3x_1 + 4x_2 \leq 80$$
$$-3x_1 + 4x_2 \geq 8$$
$$x_1 + 4x_2 \geq 40$$
$$x_1, x_2 \geq 0$$

(10 marks)

b) Suppose in a) above the left hand side values changed from 80 to 60, 8 to 10, and 40 to 50 respectively, determine the new level of optimum production. (6 marks)

c) What would happen if the constraint $2x_1 + 8x_2 \geq 14$ is added to the LP problem. (4 marks)

QUESTION FOUR (20 MARKS)

a) Use the Two-phase method to solve the following LP problem

Minimize $Z = 12x_1 + 14x_2 + 15x_3$

Subject to

$$4x_1 + 8x_2 + 6x_3 \geq 64$$

$$3x_1 + 6x_2 + 12x_3 \geq 96$$

(8 marks)

$$x_1, x_2 \geq 0$$

b) A company has three ware houses A, B and C and four stores W, X, Y and Z. The warehouses have altogether a surplus of 1600 units of a given commodity as follows

A	300
B	900
C	400

The four stores together need a total of 1600 units of the commodity as follows

W	400
X	600
Y	500
Z	100

The cost of transporting one unit in Ksh from each warehouse to store is shown in the table below

	W	X	Y	Z
A	400	200	600	400
B	500	700	900	200
C	350	820	530	680

Determine which method would be cheaper and by how much between Vogel's approximation and the least cost cell method as far as the cost of transport is concerned (12 marks)

QUESTION FIVE (20 MARKS)

a) Consider the Linear Programming Problem

Maximise $Z = c_1x_1 + c_2x_2 + c_3x_3$

Subject to

$$\begin{pmatrix} a_{11} \\ a_{21} \\ a_{31} \\ a_{41} \end{pmatrix} x_1 + \begin{pmatrix} a_{11} \\ a_{21} \\ a_{31} \\ a_{41} \end{pmatrix} x_2 + \begin{pmatrix} a_{11} \\ a_{21} \\ a_{31} \\ a_{41} \end{pmatrix} x_3 + \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} s_1 + \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} s_2 + \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix} s_3 + \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix} s_4 = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{pmatrix}$$

$x_1, \dots\dots\dots\dots s_4 \geq 0$

In the process of solving the problem, a Tableau appears as follows

Basis	x_1	x_2	x_3	S_1	S_2	S_3	S_4	B
Z	0	-140	0	100	0	0	160	35400
x_3	0	$-\frac{1}{2}$	1	1	0	0	$-\frac{1}{2}$	18
S_2	0	$\frac{1}{2}$	0	$-\frac{1}{2}$	1	0	0	11
S_3	0	2	0	$-\frac{1}{2}$	0	1	$-\frac{1}{2}$	16
x_1	1	1	0	$-\frac{1}{2}$	0	0	$\frac{1}{2}$	31

i) Determine the solution to the linear programming problem

ii) Find the values of

$$\begin{pmatrix} a_{11} \\ a_{21} \\ a_{31} \\ a_{41} \end{pmatrix}, \begin{pmatrix} a_{12} \\ a_{22} \\ a_{32} \\ a_{42} \end{pmatrix}, \begin{pmatrix} a_{13} \\ a_{23} \\ a_{33} \\ a_{42} \end{pmatrix} and \begin{pmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{pmatrix}$$

iii) Find the values of C_1 , C_2 and C_3 (14 marks)

b) Form a dual of the primal problem in a) above given an additional constraint as $2x_1 + x_2 - 4x_3 = 120$ (6 marks)