JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY

SCHOOL OF MATHEMATICS AND ACTUARIAL SCIENCE

FIRST YEAR FIRST SEMESTER EXAMINATION FOR THE DEGREE OF MASTERS OF SCIENCE IN PURE AND APPLIED MATHEMATICS

COURSE CODE: SMA 807
COURSE TITLE: COMPLEX ANALYSIS I

DATE: 28/02/2013 **TIME: 2.00-5.00PM**

INSTRUCTIONS:

1. This examination paper contains six questions. Answer **any four questions**.
2. Start each question on a fresh page.
3. Indicate question number clearly at the top of each page.

QUESTION ONE (15 marks)

a) Evaluate the line integral $\int_0^{1+i}\left(x-y+ix^2\right)dz$ along the imaginary axis from $z=0$ to $z=i$ and along a line parallel to real axis from $z=i$ to $z=1+i$ (5 marks)

b) If $f(z)$ is an analytic function in the upper half of the z - plane and $a=\xi+i\eta$ is any point in this upper half plane, show that $f(a)=\frac{1}{\pi}\int_{-\infty}^{\infty}\frac{\eta f(z)}{(x-\xi)^2+\eta^2}dx$ (10 marks)

QUESTION TWO (15 marks)

a) Find the Laurent series for $f(z)=\frac{1}{z(z-1)}$ for $0<|z|<1$ (6 marks)

b) Given that $f(z)$ is analytic at all points inside and on a simple closed curve C, except at a finite number of isolated singular points within C, prove that $\oint_c f(z)dz=2\pi i$ (residues at singular points within C) (9 marks)

QUESTION THREE (15 marks)

a) Evaluate $\oint_c\frac{3z^2+z}{z^2-1}dz$, where C is the circle $|z-1|=1$ (6 marks)

b) Prove that if $f(z)$ is analytical within and on closed curve C and a is any point within C, then

$f(a)=\frac{1}{2\pi i}\oint_c\frac{f(z)}{z-a}dz$ (10 marks)

QUESTION FOUR (15 marks)

a) Use Rouche's Theorem or otherwise to show that all the roots of $P(z)=z^8-4z^3+10$ lie between $1\le|z|\le 2$ (5 marks)

b) Evaluate the integral $\int_0^{\infty}\frac{dx}{x^4+1}$ (10 marks)

QUESTION FIVE (15 marks)

a) Show that when $|z+1|<1$, then $z^{-2}=1+\sum_{n=1}^{\infty}(n+1)(z+1)^n$ (5 marks)

b) Find a function harmonic in the upper half of the z - plane, which takes the following values on th x axis:

$$G(x)=\begin{cases}1, & x<-1\\ 0, & -1<x<1\\ -1 & x>1\end{cases}$$

QUESTION SIX (15 marks)

Evaluate by the method of complex variables, the integral

$$\int_{-\infty}^{\infty}\frac{x^2}{\left(1+x^2\right)^3}dx$$