BONDO UNIVERSITY COLLEGE
UNIVERSITY EXAMINATIONS 2012
FIRST YEAR FIRST SEMESTER EXAMINATIONS
MASTER OF SCIENCE IN APPLIED MATHEMATICS
SMA 817: ORDINARY DIFFERENTIAL EQUATIONS I

INSTRUCTION: Answer any THREE questions.

QUESTION ONE (20 MARKS)

a) Solve the differential equations

i) $\left(1-x^2\right)\frac{dy}{dx}+xy=x\left(1-x^2\right)\sqrt{y}$ (7 marks)

ii) $\frac{d^2y}{dx^2}-3\frac{dy}{dx}+2y-5e^x=0$ (6 marks)

b) Show that for a second order differential equation of the form $a\frac{d^2y}{dx^2}+b\frac{dy}{dx}+cy=f(x)$ for $f(x)\neq 0$ and taking $y=c_1u_1(x)+c_2u_2(x)$ to be a solution of $a\frac{d^2y}{dx^2}+b\frac{dy}{dx}+cy=0$, then by replacing arbtrary constants c_1 and c_2 by $v_1(x)$ and $v_2(x)$ then we could solve the pair of simultaneous equations

$$\begin{aligned} v_1'u_1+v_2'u_2&=0\\ v_1'u_1'+v_2'u_2'&=f(x) \end{aligned}$$ (7 marks)

To obtain the solution to the particular integral $y=u_1v_1+u_2v_2$

QUESTION TWO (20 MARKS)

a) Show that $u_1=e^x$ is a solution to the differential equation $(x-1)\frac{d^2y}{dx^2}-x\frac{dy}{dx}+y=0$, hence use the reduction of order method to find the second linearly independent solution $u_2(x)$ (8 marks)

b) Find all the solutions of the equation $\dot{X}=\begin{pmatrix}1&-1&4\\3&2&-1\\2&1&-1\end{pmatrix}\vec{X}$ (12 marks)

QUESTION THREE (20 MARKS)

Given the system of first order ordinary differential equations

$$\frac{dx}{dt} = 5x + y + 3z$$

$$\frac{dy}{dt} = x + 7y + z$$

$$\frac{dz}{dt} = 3x + y + 5z$$

a) Express the system in the matrix form $\dot{\underline{X}} = A\underline{X}$ (2 marks)

b) Show that $\underline{u} = [1,1,1]^t$, $\underline{v} = [-1,0,1]^t$ are eigenvectors of A (7 marks)

c) Determine $\Phi(t)$, the fundamental matrix of the system (6 marks)

d) Obtain $\underline{X}$ the general solution of the system (5 marks)

QUESTION FOUR (20 MARKS)

Given the system of nonlinear differential equations

$$\frac{dx}{dt} = -2xy$$

$$\frac{dy}{dt} = -x + y + xy - y^2$$

(a) Find all its critical points (8 marks)

(b) Determine the stability nature of each of the critical points in part (a) (12 marks)

QUESTION FIVE (20 MARKS)

a) Prove that if $x_1(t)$ and $x_2(t)$ are linearly independent on $L(x) = 0$ on an interval I then the wronskian $W[x_1(t), x_2(t)] \neq 0$ (6 marks)

b) Find e^{At} if $A = \begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & -2 \\ 3 & 3 & 1 \end{pmatrix}$ (14 marks)