JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY
SCHOOL OF MATHEMATICS ,ACTUARIAL SCIENCE AND BPS

UNIVERSITYDRAFT EXAMINATION FOR BSc/BEd IN MATHEMATICS

SUPPLEMENTARY/SPECIAL

4th YEAR 1st SEMESTER 2019/2020ACADEMIC YEAR

MAIN CAMPUS

COURSE CODE: SMA405
COURSE TITLE: PARTIAL DIFFERENTIAL EQUATIONS I

EXAM VENUE: AUDITORIUM STREAM: BSc Y4S1

TIME: 2 HOURS EXAM SESSION:

Instructions:

Answer question1 and any other two questions

1. **Show all the necessary working**
2. **Candidates are advised not to write on the question paper**
3. **Candidates must hand in their answer booklets to the invigilator while in the examination room**

.

Question 1:(30MKS) COMPULSORY

(a) Given the function
$F(x,y) = 4x^2y - y^2 - 8x^2 - 2x^4 + 10$

(i).Find $\dfrac{\partial F}{\partial x}$, $\dfrac{\partial F}{\partial y}$,

(ii) Determine all the stationary points of F

(iii) Find $\dfrac{\partial^2 F}{\partial x^2}$, $\dfrac{\partial^2 F}{\partial y^2}$ $\dfrac{\partial^2 F}{\partial x \partial y}$

(iv) Determine the nature of the stationary points of F (18mks)

(b) Given the partial differential equation

(i) $x\dfrac{\partial F}{\partial x} - y\dfrac{\partial F}{\partial y} = 3xy^2$

(ii) $x^2\dfrac{\partial^2 F}{\partial x^2} - y^2\dfrac{\partial^2 F}{\partial y^2} + x\dfrac{\partial F}{\partial x} - y\dfrac{\partial F}{\partial y} = 0$

(iii) $x^2\dfrac{\partial^3 F}{\partial x^3} - y^2\left(\dfrac{\partial^2 F}{\partial y^2}\right)^4 + x\dfrac{\partial F}{\partial x} - y\dfrac{\partial F}{\partial y} = 0$

State in each the ; ORDER, DEGREE and whether **LINEAR or NONLINEAR. (12mks)**

.

Question 2:(20MKS) Solve the homogeneous the partial differential equation

(i) $\left(D^2 - DD' - 6D'^2\right)u = 0$

(ii) $\left(4D^2 - 12DD' + 9D'^2\right)u = 0$

Question 3. :(20MKS)

Solve the inhomogeneous the partial differential equation

(i) $\left(D^2 - 3DD' - 4D'^2\right)u = e^{x+2y}$

(ii) $\left(D^2 - DD' - 6D'^2\right)u = \sin x \cos 2y$

Question 4: **:(20MKS)**

Solve the partial differential equation $x^2\dfrac{\partial^2 F}{\partial x^2} - y^2\dfrac{\partial^2 F}{\partial y^2} + x\dfrac{\partial F}{\partial x} - y\dfrac{\partial F}{\partial y} = 0$

Use the change of variables from x , y to u , v where $u = xy$, $v = \dfrac{y}{x}$

Question 5: :(20MKS)

Consider a perfectly flexible elastic string ,stretched between two points at $x = 0$ and $x = 1$ with uniform tension τ .

If the string is displaced slightly from its initial position while the ends remain fixed , and then released , the string will oscillate . The position P in the string at any instant will then be a function of its distance from one end (x,) of the string and also of time (t) i.e. $u = u(x,t)$,

. The equation of the motion is given by the partial differential equation

$$\frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 u}{\partial t^2} \quad .$$

Using variable separation, of the form $u(x,t) = X(x)T(t)$

(a) Show that the variables X , T satisfy the ordinary differential equations

$$\frac{1}{X}\frac{d^2X}{dx^2} = \frac{1}{T}\frac{d^2T}{dt^2}$$

(b) determine the displacement $u(x,t)$ given

the boundary conditions

$u(0,t) = u(1,t) = 0$ for all time $t \geq 0$

and the initial condition

$u(x,0) = 0$