Abstract

The notion of similarity orbits($S(T)$) of Hilbert space operators have been extensively investigated by many researchers such as Herrero, Fialkow and Hadwin among others. Various properties of $S(T)$ such as closure and compact perturbation have been proven. However, if we regard similarity orbit as a set consisting of operators on which a topology has been assigned to become a topological space, limited has been done in terms of their characterizations. Therefore, investigating invariant sets of similarity orbit of norm-attainable operators is interesting particularly, with regard to the invariant subspace problem. The objectives of this study were to: Investigate the denseness of similarity orbits of invariant subspaces of norm-attainable operators; investigate the compactness of similarity orbits of invariant subspaces of norm-attainable operators and; establish the necessary and sufficient conditions for stability of similarity orbit of invariant subspaces of norm-attainable operators. The methodology involved the concept of convergence and De Morgan's laws to investigate the denseness and compactness of similarity orbit. In addition, we also employed the technical approach of Jordan canonical form, tensor product and the theory of spectra of linear operators to determine the stability of similarity orbits. The results obtained show that; a set is dense if the intersection of two open sets is nonempty and denseness is transitive. In addition, closedness implies compactness, union and intersection of compact sets is compact and compactness is independent on the space the set is embedded among others. Lastly, similarity orbit is stable if its norm and spectral radius is less than one. In conclusion, we characterized similarity orbit in terms of denseness and compactness and investigated it's stability. The results obtained are contribution in the field of functional analysis(particularly, $S(T)$) and a motivation to a further research.