Abstract

The study of the *Davis-Wielandt shell* forms a very important generalization of the numerical range in functional analysis. Hiroshi Nakazito and Mao-Ting Chien studied the connections between the *q-numerical range* and the Davis-Wielandt shell. Chi-Kwong Li and Yiu-Tung Poon studied the boundary of the Davis-Wielandt shells of normal operators. However, the characterization of the *essential numerical range*, $W_e(T)$, and the *Davis-Wielandt shell*, $DW(T)$ has not been exhausted. One of the pending questions that remained was: What are the connections between the $W_e(T)$ and the $DW(T)$ of an operator? Moreover, what are the conditions when $W_e(T)$ and the classical numerical range, $W(T)$, coincide in the *Davis-Wielandt shell*? Therefore we have presented the *Davis-Wielandt shells* and the *essential numerical range* of operators in Hilbert spaces. In this study, we have investigated the following; the relationship between the $DW(T)$ of an operator and the $W_e(T)$; the relationship between the *essential spectrum* and the $DW(T)$ of an operator; the condition when the $W_e(T)$ and the $W(T)$ coincide in the *Davis-Wielandt shell*. The methodology involved the use of inner product spaces, the Cauchy-Schwarz and triangle inequalities. The results of this study showed that the *essential numerical range* and the *Davis-Wielandt shells* of an operator share a variety of properties, for instance, identity property and unitary invariance. It was also noted that the essential spectrum is contained in the closure of the first co-ordinate of the *Davis-Wielandt shell*. Moreover, the $W_e(T)$ and the $W(T)$ coincide in $DW(T)$ if and only if $T = \lambda I$. The results obtained would be useful in applications involving systems of differential equations and aerodynamics.