Abstract

Hilbert space operators have been studied by several mathematicians over decades and various interesting results have been obtained. There are several classes of Hilbert space operators, for example, normal, positive, completely positive, paranormal, hyponormal and norm-attainable operators among others. Some of the properties of these operators that have been studied include norms, numerical ranges, spectra, boundedness and orthogonality. On norm-attainable operators, characterization has been done though not exhausted but not much has been done with regards to their norms. The unsolved problem has been to establish the necessarily and sufficient conditions for $\|\mathcal{U}_{\mathcal{N},\mathcal{A},\mathcal{B}} \mid B(H)\| \geq \|A\|\|B\|$ to hold if H is taken to be infinite dimensional nonseparable Hilbert space. The objectives of the study are to: characterize norm-attainable operators on infinite dimensional and nonseparable Hilbert spaces, determine norm estimates for norm-attainable elementary operators on infinite dimensional and nonseparable Hilbert spaces and further determine norm estimates of norm-attainable elementary operators on Banach algebras. The methodology involved the techniques of inner products, tensor products, numerical ranges, arithmetic geometric mean and some known mathematical inequalities like Cauchy-Schwartz inequality and triangle inequality.The results shows that $\|\mathcal{M}_{P,Q}(X)\| = \|P\|\|Q\|$ and $\|\mathcal{U}_{P,Q} \mid \mathcal{E}[NA(H)]\| \geq 2\|Q\|\|P\|$. The results obtained maybe useful in applications in operator theory particularly operator algebras.