ABSTRACT

A mixed distribution can be obtained when two or more distributions are combined. The integral of the product of the mixture is determined for continuous case and for the discrete the sum of the product is determined. Binomial distribution is discrete distribution while beta distribution is continuous distribution. However the joint distribution of their mixture turns out to be in the form of beta distribution which is continuous. McDonald generated many generalized beta distributions that could be applicable as mixing distributions in the Binomial mixtures. Bowman and other researchers used the transformation $p = e^{-\theta}$ to generate a large number of generalized mixing distributions. The most common prior distribution for the probability of success is classical beta distribution. However beta II distribution and its generalizations have not been used as prior distributions. Also the methods of construction of the mixtures such as direct integration and method of moments were not proved to be identical. Studies in the statistical literature concentrated on binomial mixtures but did not focus on posterior distributions and Bayesian inference. This study focused on Beta-binomial mixture whose origin dated back to the year 1948 when Skellam mixed a binomial distribution with its parameters being probability of success taking beta distribution. The objective was to construct binomial mixtures using beta prior distribution and its generalizations. Specifically it focused on proving identity of two methods of construction of binomial mixtures, construction of the binomial mixtures with parameterized beta distribution such as $p = \frac{\theta}{1+\theta}$ and $p = e^{-\theta}$, differential equations of binomial mixtures using Panjer-Willmot and Hesselager's recursive models and posterior distributions were obtained. Construction of the beta-binomial mixtures involved use of special functions such as Gamma function, beta function, confluent hypergeometric function and Gauss hypergeometric function. Transforms such as generating functions and laplace transforms were applied to construct probability distributions. To obtain differential equations the recursive models obtained from the Beta-Binomial mixtures were fitted to appropriate known models such as Panjer-Willmot and Hesselager's recursive models. This study is important as it addresses the problem of over-dispersion, a challenge which is common in modeling data that has binomial outcomes. This was achieved as the variance of the binomial mixtures obtained is theoretically larger and hence more likely to fit the variance of the empirical data. The mixture was applied to group screening design where the varying probability of contamination was estimated, with the outcome in support of Bayesian technique than Maximum likelihood technique for low values of p and vise versa for higher values of p.