JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY

SCHOOL OF BIOLOGICAL PHYSICAL MATHEMATICS AND ACTUARIAL SCIENCES

UNIVERSITY EXAMINATION FOR DEGREE OF BACHELOR OF SCIENCE ACTUARIAL

SPECIAL RESITS DECEMBER 2022

MAIN REGULAR

COURSE CODE: WMB 9108

COURSE TITLE: CALCULUS I

EXAM VENUE: **STREAM: (Bed/BSc. Actuarial)**

DATE: **EXAM SESSION:**

TIME: 2.00 HOURS

Instructions:

1. **Answer question 1 (Compulsory) and ANY other 2 questions**
2. **Candidates are advised not to write on the question paper.**
3. **Candidates must hand in their answer booklets to the invigilator while in the examination room.**

QUESTION ONE (COMPULSORY) (30 marks)

a) State any four properties of limits (4 marks)

b) Differentiate from the first principle $f(x) = x^2 + 4x$ (5 marks)

c) Establish that:

$$\lim_{x \to 0} \frac{\sin x}{x} = 1$$ (5 marks)

d) Determine the point of discontinuity (if any) of the function $f(x)$

$$f(x) = \frac{x^2 + 3x - 1}{x - 3}$$

If the discontinuity is removable, define the function to make it continuous. (5 marks)

e) Evaluate $\lim_{x \to \infty} \frac{x+4}{2x+2}$ (5 marks)

f) The parametric equations of a curve are $x = t^3$, $y = 2t^3 - t$, find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$

(6 marks)

QUESTION TWO (20 marks)

a) Find the gradient of the curve $y = \ln\sqrt{1 + \sin 2x}$ at the point where $x = \frac{\pi}{2}$ (5 marks)

b) Find the limit (if it exists)

$$\lim_{x \to 0} \frac{\sqrt{x^2 + 4} - 2}{x^2}$$ (5 marks)

c) Differentiate the following with respect to $xy = e^{\sqrt{\cos x}}$ (5 marks)

d) For what values of a and b is $f(x) = \begin{cases} -2, & x \le 1 \\ ax - b, & -1 < x < 1 \\ 3, & x \ge 1 \end{cases}$

Continuous at every x ? (5 marks)

QUESTION THREE (20 marks)

a) Find the derivatives of the following functions $k(x) = \tan(6x^3 - 4x^2)\cot x$ (5marks)

b) Find the equation of the tangent and normal to $3y^2 - 4x^2 = 9$ at the point (a, b) on the curve

(7marks)

c) If $y = \frac{\sin x}{x}$, find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ and prove that $x^2\frac{d^2y}{dx^2} + 4x\frac{dy}{dx} + (x^2+2)y = 0$ (8marks)

QUESTION FOUR (20 marks)

a) If $y = \cos^{-1}(\sin x)$, show that $\frac{dy}{dx} = -1$ (6marks)

b) Evaluate $\frac{d}{dx}\left(\ln\frac{\cos x}{\sqrt{4-3x^2}}\right)$. (6 marks)

c) Sketch the curve $y = (x+1)(x^2+2x-8)$ giving all significant point (8 marks)

QUESTION FIVE (20 marks)

a) Show that given that $u = u(x)$ and $v = v(x)$, then if $y = uv$, then $\frac{dy}{dx} = v\frac{du}{dx} + u\frac{dv}{dx}$ hence solve $y = \cos(x+1)\sin(x+1)$ (10 marks)

b) Flepy Kenya, a subsidiary of Flet Snacks, produces Doritos Cheetos, Fritos corn chips, and a variety of other salty, sweet, or grain-based snacks. Based on data from 2009 to 2019, the net sales (revenue) of Flepy Kenya may be modeled by

$$R(t) = -168t^2 + 907t + 8232$$

million Kenya shillings and the operating profit (earnings before interest and taxes) may be modeled by

$$P(t) = -47.5t^2 + 283.5t + 1679$$

million Kenya shillings, where t is the number of years since 2009.

i) In what year are the net sales projected to reach maximum? (3 marks)

ii) Find the cost function for Flepy Kenya. (2 marks)

iii) According to the model, in what year are costs expected to reach a maximum? (3 marks)

iv) Compare the results of (i) and (iii). Do the results seem reasonable? (2 marks)