JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY

SCHOOL OF BIOLOGICAL PHYSICAL MATHEMATICS AND ACTUARIAL SCIENCE

UNIVERSITY EXAMINATION FOR DEGREE OF BACHELOR OF EDUCATION

2ND YEAR 1ST SEMESTER 2023/2024 ACADEMIC YEAR

REGULAR (MAIN)

COURSE CODE: WAB 2208

COURSE TITLE: PROBABILITY AND DISTRIBUTION THEORY I

EXAM VENUE: **STREAM: ED. SCIENCE/ARTS/SNE**

DATE: **EXAM SESSION:**

TIME: 2.00 HOURS

Instructions:

1. **Answer Question ONE and ANY other two questions**
2. **Candidates are advised not to write on the question paper.**
3. **Candidates must hand in their answer booklets to the invigilator while in the examination room.**

QUESTION ONE (COMPULSORY)-(30 MARKS)

a) The joint density function of two continuous random variables X and Y is given by

$$f(x,y) = \begin{cases} k(2y - x), & 0 \le x \le 1, 0 \le y \le 2 \\ 0, & otherwise \end{cases}$$

Obtain the value of k [7 Marks]

b) Let X be a gamma distribution with the probability density function;

$$f(x) = \begin{cases} \frac{1}{32} x e^{-\frac{x}{4}}, & x > 0 \\ 0, & otherwise \end{cases}$$

.

Obtain the mean of X. [8 Marks]

c) The proportion of time Y that a sheet metal stamping machine is down for repair follows a Beta distribution $f(y) = \begin{cases} 6y(1-y), & 0 < y < 1 \\ 0, & otherwise \end{cases}$.

Obtain the probability that the sheet stamping machine will be down for repair for more than 50% the time allocated for repair. [7 Marks]

d) The joint probability mass function of two discrete random variables X and Y is given by

$$f(x,y) = \begin{cases} \frac{1}{48}(2x + y), x = 0,1,2,3; y = 0,1,2 \\ 0, \quad otherwise \end{cases}$$

Find the marginal densities of X hence the conditional probability $\mathrm{P}(\mathrm{y} = 1/\mathrm{x} = 2)$

[8 Marks]

QUESTION TWO (20 MARKS)

a) Let $f(x,y) = \begin{cases} 6y, & 0 < y < x < 1, \ y > 0, x > 0 \\ 0, & otherwise \end{cases}$

Show that $f(x,y)$ is a joint probability density function. [6 Marks]

b) Let X be a random variable with the density function

$$f(x) = \begin{cases} 2x, & 0 < x < 1 \\ 0, & otherwise \end{cases}$$

Obtain the density function of a new random variable U where $\mathrm{u} = 8 - \mathrm{x}^2$ [6 Marks]

c) Let X be a Chi square random variable with paramater ϑ and the density function given by

$$f(x) = \begin{cases} \dfrac{x^{\frac{\vartheta}{2}-1} e^{-x/2}}{2^{\frac{\vartheta}{2}} \Gamma\left(\frac{\vartheta}{2}\right)}, & x > 0, \\ 0, & otherwise \end{cases}$$

It is known that the mean of this distribution is ϑ. By derivation, obtain an expression for the variance of the distribution. [8 Marks]

QUESTION THREE (20 MARKS)

a) Two discrete random variables X and Y have the joint probability function given by

	$Y = 0$	$Y = 1$	$Y = 2$
$X = 1$	1/12	3/12	1/12
$X = 2$	2/12	1/12	1/12
$X = 3$	1/12	1/12	1/12

Obtain

i. The marginal distributions of X and Y [4 marks]

ii. $P(Y \leq 1)$ [2 marks]

iii. $f(X/Y = y)$ [6 marks]

b) A random variable X can be modeled as exponential with mean θ. Suppose it is known that $P(X < 10) = 0.3935$. Find θ, the mean of this distribution to the nearest whole number. [8 marks]

QUESTION FOUR (20 MARKS)

The random variables X and Y have joint p.d.f given by

$$f(x, y) = \begin{cases} \dfrac{4}{81} xy, & 0 < x < 3, 0 < y < 3 \\ 0, & otherwise \end{cases}$$

Obtain

i. The means of X and Y: E(X), E(Y) [4 Marks]

ii. The variances of X and Y: Var(X), Var(Y) [8 Marks]

iii. The joint expectation E(XY) [4 Marks]

iv. Cov (XY) [3 Marks]

v. Are X and Y independent? [1 Mark]

QUESTION FIVE (20 MARKS)

The joint p.d.f of three continuous random variables X , Y and Z is defined as follows

$$f(x,y,z) = \begin{cases} k(xy+z), & 0 < x < 2\ , 0 < y < 2, 0 < z < 1 \\ 0, & otherwise \end{cases}$$

Calculate:

i. the value of k ,

ii. the marginal distribution of Z hence the mean of Z [20 Marks]