JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY

SCHOOL OF BIOLOGICAL PHYSICAL MATHEMATHICS AND ACTUARIAL SCIENCE

UNIVERSITY EXAMINATION FOR DEGREE OF BACHELOR OF SCIENCE IN ACTUARIAL SCIENCES

4TH YEAR 1ST SEMESTER 2023/2024 ACADEMIC YEAR

REGULAR (MAIN)

COURSE CODE: WAB 2415

COURSE TITLE: FURTHER DISTRIBUTION THEORY

EXAM VENUE: **STREAM: (B.sc. Actuarial Science)**

DATE: **EXAM SESSION:**

TIME: 2.00 HOURS

Instructions:

1. **Answer question 1 (Compulsory) and ANY other 2 questions**
2. **Candidates are advised not to write on the question paper.**
3. **Candidates must hand in their answer booklets to the invigilator while in the examination room.**

QUESTION ONE (30 MARKS)

a) The average number of calls received per hour by an insurance company's switchboard is 5. Calculate the exact probability that in a working day of eight hours, the number of telephone calls received will be

   i. exactly 36 (2 Marks)

   ii. between 42 and 45 inclusive (3 Marks)

b) Calculate the approximate probabilities using the normal approximation in (a) above (6 Marks)

c) Use a normal approximation to calculate an approximate value for the probability that an observation from $Gamma(25,50)$ random variable falls between 0.4 and 0.8 (4 Marks)

d) What is the approximate probability that the mean sample of 10 observations from a $Beta(10,10)$ random variable falls between 0.48 and 0.52. (7 Marks)

e) The probability distribution function of a random variable X is given by $f(x)=\begin{cases} 2x & o<x<1 \\ 0 & otherwise \end{cases}$. Show that of k increases $\Pr(|X-\mu| \geq k\sigma)$ decreases. (8 Marks)

QUESTION TWO (20 MARKS)

a) Given that X is a continuous random variable, then X is said to have a chi – square distribution with probability density function given by

$$f(x)=\begin{cases} \dfrac{1}{\Gamma\left(n/2\right)2^{n/2}} x^{\frac{n}{2}-1} e^{-x/2} & x>0 \\ 0 & elsewhere \end{cases}$$

Find

   i. the moment generating function of the chi – square (8 Marks)

   ii. the mean and the variance of the chi – square distribution. (9 Marks)

b) Given that the moment generating function of a random variable X is given by $M_X(t)=(1-2t)^{-8}$, $t<1/2$

   i. State the distribution of X. (1 Mark)

   ii. Hence find the mean and variance of X (2 Marks)

QUESTION THREE (20 MARKS)

a) A random variable X is said to follow Pareto (type I) distribution with its probability density function given by

$$f(x) = \frac{\alpha k^{\alpha}}{x^{\alpha+1}} \qquad x > k,$$

where k is the scale parameter and α is the shape parameter. Obtain the mean and variance of this distribution. (12 Marks)

b) The random variable X is an insurer's annual hurricane – related no indent. Suppose that the density function of X is

$$f(x) = \frac{2.2(250)^{2.2}}{x^{3.2}} \qquad x > 250$$

Calculate the mean and median of the annual hurricane related loss. (8 Marks)

QUESTION FOUR (20 MARKS)

a) The time taken by the milkman to deliver milk to high street is normally distributed with mean of 12 minutes and a standard deviation of 2 minutes. He delivers milk every day. Estimate the number of days during the year when he takes longer than 17 minutes. (5 Marks)

b) A continuous random variable X follows a Weibull distribution with parameters β and α whose probability density function is given by

$$f(x) = \frac{\beta}{\alpha}\left(\frac{x}{\alpha}\right)^{\beta-1} e^{-\left(\frac{x}{\alpha}\right)^{\beta}} ; \qquad \beta, \alpha > 0 \qquad x > 0$$

β is the shape parameter and α is the scale parameter. Obtain the mean and variance of this distribution. (15 Marks)

QUESTION FIVE (20 MARKS)

Let X be a standard normal variable with mean of zero and a variance of one. Let U be a chi – square variable with n degrees of freedom. Given that X and U are stochastically independent, we define another random variable given by

$$T = \frac{X}{\sqrt{U/n}}$$

Determine

i. the probability distribution function of T. (12 marks)

ii. the mean and variance of T. (8 marks)