JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY

SCHOOL OF MATHEMATICS AND ACTUARIAL SCIENCE

UNIVERSITY EXAMINATION FOR DEGREE OF BACHELOR OF SCIENCE

ACTUARIAL SCIENCE

1st YEAR 1st SEMESTER 2023/2024

REGULAR (MAIN)

COURSE CODE: WAB 2101

COURSE TITLE: PRINCIPLES OF ACTUARIAL SCIENCE.

EXAM VENUE: **STREAM: (BSc Actuarial Science)**

DATE: **EXAM SESSION:**

TIME: 2.00 HOURS

Instructions:

1. **Answer question 1 (Compulsory) and ANY other 2 questions**
2. **Candidates are advised not to write on the question paper.**
3. **Candidates must hand in their answer booklets to the invigilator while in the examination room.**

QUESTION ONE [30 marks]

a. Define the following terms according to insurance contracts

i. A pure endowment

(2 Marks)

ii. An endowment assurance

(2 Marks)

iii. A term assurance

(2 Marks)

b. State and explain three examples of contingent annuities.

(6 marks)

c. i. Calculate the nominal annual interest rate convertible quarterly that is equivalent to an interest rate of 5% *pa* effective.

(2 Marks)

ii. Calculate the annual effective interest rate that is equivalent to a nominal interest rate of 12% *pa* convertible four-monthly.

(2 Marks)

d. The force of interest at time t is $\delta(t) = 0.02 + 0.01t$. Calculate the accumulated value at time 8 of an investment of £1,000 at time:

(i) 0

(3 marks)

(ii) 5

(3 marks)

e. State the Principle of correspondence

(2 marks)

f. If the age label is 'age nearest birthday', give the exact age at which a life attains age label x

(1 marks)

g. Fill the table below

Definition of x	Rate interval	$\hat{\mu}$ estimates	$\hat{q}$ estimates
Age last birthday			
Age nearest birthday			
Age next birthday			

(3 marks)

h. Calculate the present value of an annuity that pays £300 *pa* monthly in arrears forever using an annual effective rate of interest of 6%.

(2 marks)

QUESTION TWO [20 marks]

a. The force of interest at time *t* is given by:

$$\delta(t) = \begin{cases} 0.08 - 0.001t & 0 \le t < 3 \\ 0.025t - 0.04 & 3 \le t < 5 \\ 0.03 & 5 \le t \end{cases}$$

(i) Calculate the present value at time 2 of a payment of £1,000 at time 10.

(7 marks)

(ii) Calculate the annual effective rate of interest that is equivalent to this variable force of interest from time 2 to time 10.

(3 marks)

b. i) Given $\delta = 8\%$, calculate i', $i(4)$ and $d(12)$.

(ii) Given $i' = 7\%$, calculate d' , $d(4)$, $i(2)$ and δ .

(iii) Given $d' = 9\%$, calculate i', $d(2)$, $i(12)$ and δ .

(10 marks)

QUESTION THREE [20 marks]

a. A mortality investigation was held between 1 January 2016 and 1 January 2018. The following

information was collected. The figures in the table below are the numbers of lives on each census

date with the specified age labels.

Age last birthday	1 Jan 2016	1 Jan 2017	1 Jan 2018
48	3,486	3,384	3,420
49	3,450	3,507	3,435
50	3,510	3,595	3,540

During the investigation there were 42 deaths at age 49 nearest birthday. Estimate $\square_{49}$ stating any assumptions that you make.

(10 marks)

b. A researcher is studying the mortality rates of older males in a certain population over the calendar years 2016 and 2017. The researcher has obtained the following data:

- the number of males in the population at each age, classified by age next birthday, on 1 April in 2015, 2016, 2017 and 2018
- the number of deaths at each age, classified by age next birthday at the time of death.

You are given the following extract from the data:

Number of males in population

Age next birthday	1/4/15	1/4/16	1/4/17	1/4/18
81	6,010	5,980	6,130	6,200
82	5,320	5,310	5,480	5,520
83	5,680	5,800	5,750	6,030
84	5,150	5,230	5,250	5,150

Number of deaths

Age next birthday	In 2016	In 2017
81	354	348
82	375	391
83	430	432
84	442	437

Estimate $\mu_{81.5}$ using these data values.

(10 marks)

QUESTION FOUR [20 marks]

a. Using an interest rate of 12% *pa* convertible monthly, calculate:

i. the combined present value of an immediate annuity payable monthly in arrears such that payments are £1,000 *pa* for the first 6 years and £400 *pa* for the next 4 years, together with a lump sum of £2,000 at the end of the 10 years.

(3 marks)

ii. the amount of the level annuity payable continuously for 10 years having the same present value as the payments in (i).

(3 marks)

iii. the accumulated value of the first 7 years' payments at the end of the 7th year for the payments in (i) and (ii).

(4 marks)

b. A man makes payments into an investment account of \$200 at time 5, \$190 at time 6, \$180 at time 7, and so on until a payment of \$100 at time 15. Assuming an annual effective rate of interest of 3.5%, calculate:

i. the present value of the payments at time 4,

(6 marks)

ii. the present value of the payments at time 0,

(2 marks)

iii. the accumulated value of the payments at time 15.

(2 marks)

QUESTION FIVE [20 marks]

a. Using an interest rate of 6% *pa* effective and AM92 Ultimate mortality, calculate:

(i) $A_{50:\overline{15}|}^{1}$

(ii) $\bar{A}_{50:\overline{15}|}$

(10 marks)

b. A 50-year-old woman purchases a deferred annuity to provide herself with an income of £15,000 *pa*, paid annually in advance from age 70 until death.
Calculate the expected present value of the benefits from this deferred annuity, using PFA92C20
mortality, and an interest rate of 4% *pa* effective.

(5 marks)

c. A continuously payable temporary annuity is sold to a life aged exactly 40. The annuity makes payments at a rate of 5,000 *pa* until age 60 or the policyholder's earlier death.
Calculate the expected present value of the annuity payments, using AM92 Select mortality and
an interest rate of 4% *pa* effective.

(5 marks)