JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY

SCHOOL OF BIOLOGICAL, PHYSICAL, MATHEMATICS AND ACTURIAL SCIENCES

UNIVERSITY EXAMINATION FOR BACHELOR OF ACTUARIAL SCIENCE

2023/24

MAIN REGULAR

COURSE CODE: WAB 2201

COURSE TITLE: Financial Mathematics I

EXAM VENUE   **STREAM: B.Sc. Actuarial Science**

DATE:……………   **EXAM SESSION: ONE**

TIME: 2 HOURS

Instructions to the Candidate:

1. **Answer   ONE and any other two questions only.**
2. **Candidates are advised not to write on the question paper.**
3. **Candidates must hand in their answer booklets to the invigilator while in the examination room.**

QUESTION ONE: 30 Marks

a. Otieno deposits $Ksh.1000$ into account in January 1,2007. The account credits interest at an effective annual rate of 5% every December 31. The person withdraws $Ksh.200$ on January 1,2009, deposits $Ksh.100$ on January 2010 and withdraws $Ksh.250$ 0n January,1,2012. What is the balance in the account just after interest is credited on 31, December 2013? [4 Marks]

b. $Ksh.4,600$ is invested at time 0 and proceeds at time 10 is $Ksh.8200$. Calculate $A(7,10)$ if $A(0,9) = 1.8$, $A(2,4) = 1.1$, $A(2,7) = 1.32$ and $A(4,9) = 1.45$. [4 Marks]

c. Equity Bank offers an annual interest rate of 15.25% compounded semi-annually and KCB Bank offers annual interest rate of 15% compounded monthly. Which Bank offers a higher effective annual interest rate? B [4 Marks]

d. If the force of interest is given by;

$$\delta(t) = \begin{cases} 0.04 & 0 \le t < 6 \\ 0.2 - 0.02t & 6 \le t \end{cases} ,$$

   i. Find the expression for the present value from time 1 to time t [4 Marks]
   ii. Find the accumulated value at time 8 of a payment of Ksh.400 at time 3
           [4 Marks]

e. Prove Algebraically that:

$$\ddot{a}_{\overline{n}\neg} = a_{\overline{n-1}\neg}$$ [5 Marks]

f. Find the present value as at 1 January 2019 of series of 100 payable the first day of each month during 2020,2021 and 2022 assuming an effective rate of interest of 8% per annum. [5 Marks]

Question 2: 20 Marks

a. Show that:

$$\bar{a}_{\overline{n}} = \frac{1-V^n}{\delta} \quad \text{if } \delta \neq 0 \qquad \text{[5 Marks]}$$

b. Find the present value of a series of payment of Ksh.1 each payable for 1 year at the beginning of each week, assuming an effective rate of interest of 8% per annum. (Assume 52.18 weeks a year). [6 Marks]

c. Cooperative Bank pays 10% effective annual interest rate over 5 years. Find the equivalent:

    i.   Effective monthly interest rate [3 Marks]

    ii.   Effective biennial interest rate [3 Marks]

    iii.   Simple annual discount rate [3 Marks]

Question 3: 20 Marks

a. Given a Force of interest;

$$\delta(t) = \begin{cases} 0.08 - 0.001t & 0 \leq t < 3 \\ 0.025t - 0.004 & 3 \leq t < 5 \\ 0.3 & 5 \leq t \end{cases}$$

    i.   Calculate the present value at time 2 of a payment of Ksh.1000 at time 10 [5 Marks]

    ii.   Calculate the annual effective rate of interest from time 2 to 10 equivalent to the force of interest function above. [3 Marks]

    iii.   Find the expression for the accumulated value from time 0 to t [8 Marks]

b. Discount Ksh.10,000 for 3 years using simple discount rate of 5% per annum. [4 Marks]

Question 4: 20 Marks

a. A student from JOOUST has won a prize and is given either:
   - Lumpsum sum of Ksh.100,000 to invest now or
   - Ksh.55,000 to invest in one year's time and Ksh.55,000 to invest in two years' time.

   If all investment is assumed to earn interest at a rate of 7% per annum effective,

   determine which option should he choose if he intends to withdraw the money after 4

   years [5 Marks]

b. A 182-day treasury bill redeemable at 100, was purchased for 96.56 at the time of issue and later sold to another investor for 8 who held the bill to maturity. The rate of return received by the initial purchaser was 4% per annum effective.

   i. Calculate the length of time in days for which the initial purchaser held the bill. [3 Marks]
   ii. Calculate the annual simple rate of return achieved by the second investment. [3 Marks]
   iii. Calculate the annual effective rate of return achieved by the second investor. [3 Marks]

c. Given; $\delta = 8\%$,
   Calculate;
   i. i [2 Marks]
   ii. $i^{(4)}$ [2 Marks]
   iii. $d^{(12)}$ [2 Marks]

Question 5: 20 Marks

a. A life insurance starts issuing a new type of 10-year saving policy to young investment who pay weekly premium of 10. The insurer assumes that it will sell 10,000 policy events over each year and that no policyholder will stop paying after taking out policy. Calculate the total premium income that will be received during the first 3 years, assuming that that are 52.18 weeks in each year. [6 Marks]

b. A company expects to receive a continuous cashflow for the next 5 years when the rate of payment is 100×0.8^t at time t years. Calculate the present value of the cashflow assuming a constant force of interest of 8% per annum throughout the period. [5 Marks]

c. Calculate:

$$a^{(12)}{}_{\overline{3.5}|} \text{ given that } i = 1.5618\% \qquad [3 \text{ Marks}]$$

d. X denotes the present value of an annuity consisting of payments of 2,000 payable at the end of each of the next 8 years, valued using an interest rate of 8% per annum convertible quarterly. Y denotes the present value of an annuity consisting of payment of 4,000 payable at the end of every fourth year for the next 16 years valued using an interest rate of 8% per annum convertible half yearly. Calculate the ratio X/Y.

[6 Marks]