On numerical radius attainability for normal self-adjoint operators

The numerical range and numerical radius are very useful in studying linear operators acting on Hilbert spaces. The operators include normal, hyponormal, normaloid, transaloid, self-adjoint, subnormal, compact and unitary. Norm attaining and numerical radius attaining operators have been studied by many mathematicians including Acosta, Paya, Ruiz, and Cardassi among others. The question of whether the numerical radius is attainable for normal operators has not yet been fully investigated, especially for self-adjoint operators. The question arising from Kittaneh is whether the numerical radius is attainable for normal self-adjoint operators. If so, what are the necessary and sufficient conditions for numerical radius attainability for normal self-adjoint operators? The objective of this paper is to establish necessary and sufficient conditions for numerical radius attainability for normal self-adjoint operators. We have the results on conditions for numerical radius attainability and denseness for Hilbert space operators. We have considered normal and self-adjoint operators. The methodology involved the use of the technical approach of tensor products in our calculations. The obtained results are important in investigating the relationship between numerical radius attainable normal operators and other norm attainable operators.