On norm preserving conditions for local automorphisms of commutative banach algebras

The history of commutative algebra first appeared in 1890 by David Hilbert which was then followed by Banach spaces in 1924 since localization reduces many problems of geometric special case into commutative algebra problems of local ring. So far, many studies on preserver problems have been focusing on linear preserver problems (LPPs) especially LPPs in matrix theory. Also in consideration has been the characterization of all linear transformation on given linear space of matrices that leave certain functions, subsets and relations invariant. Clearly, we also have spectrum preserver problem or transmission. Kadison and Sourour have also shown that the derivation of local derivation of Von Neumann algebra R are continuous linear maps if it coincides with some derivation at each point in the algebra over C. We employ the concept of 2-local automorphisms introduced by SermI that if we let A be an algebra, then the transformation $\phi: A \rightarrow \phi$ A transformation $\phi: A \to A$ is called a 2-local automorphism if for all x, $y \in A$ there is an automorphism ϕ . (xy) of A for which $\phi(x) = \phi_{x,y}(x)$ and $\phi(y) = \phi_{x,y}(y)$. In this paper, we characterize commutativity of local automorphism of commutative Banach algebras, establish the norm preserver condition and determine the norms of locally inner automorphisms of commutative Banach algebras. We use Hahn-Banach extension theorems and the great ideas developed by Richard, and Sorour to develop the algebra of local automorphisms, then integrate it with norm preserver conditions of commutative Banach algebras. The results of this work have a great impact in explaining the theoritical aspects of quantum mechanics especially when determining the distance of physical quantities.