JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY

SCHOOL OF MATHEMATICS AND ACTUARIAL SCIENCE

UNIVERSITY EXAMINATION FOR DEGREE OF MASTER OF SCIENCE

IN APPLIED MATHEMATICS

1ST YEAR 1ST SEMESTER 2017 ACADEMIC YEAR

KISUMU CAMPUS

COURSE CODE: SMA 839

COURSE TITLE: NUMERICAL ANALYSIS I

EXAM VENUE: **STREAM: (BSc. Actuarial)**

DATE: **EXAM SESSION:**

TIME: 3.00 HOURS

Instructions:

1. **Answer ANY 3 questions**
2. **Candidates are advised not to write on the question paper.**
3. **Candidates must hand in their answer booklets to the invigilator while in the examination room.**

QUESTION ONE (20 MARKS)

a) Given

$P_n(x) = f_i^{(0)} + (x - x_0) f_i^{(1)} + (x - x_0)(x - x_1) f_i^{(2)} + \ldots + (x - x_0)(x - x_1)\ldots(x - x_{n-1}) f_i^{(n)}$

verify that $P_n(x)$ passes exactly through the data points x_0, x_1 and x_2. (6 marks)

b) Consider the six points table which satisfy the function $f(x) = \frac{2}{x} + x^2$.

x	f(x)
0.4	5.1600
0.6	3.6933
0.8	3.1400
1.0	3.0000
1.2	3.1067
1.4	3.3886

(i) Give the nth degree Lagrange polynomial $P_n(x)$ which passes through $n+1$ points. Hence use it to find $P_3(0.7)$. (7 marks)

(ii) Form six-place difference table for the above data points and use it to calculate $P_3(0.7)$ by the Newton backward-difference polynomial. (7 marks)

QUESTION TWO (20 MARKS)

a) Give Gauss elimination procedure in a format suitable for computer programming. (8 marks)

b) Solve the following system of linear algebraic equations by applying Gauss elimination with partial pivoting

$$
\begin{array}{rcrcrcrcr}
2x_1 & - & 2x_2 & + & 2x_3 & + & x_4 & = & 7 \\
2x_1 & - & 4x_2 & + & x_3 & + & 3x_4 & = & 10 \\
-x_1 & + & 3x_2 & - & 4x_3 & + & 2x_4 & = & -14 \\
2x_1 & + & 4x_2 & + & 3x_3 & - & 2x_4 & = & -1
\end{array}
$$

(12 marks)

QUESTION THREE (20 MARKS)

Find the solution of the following system of linear algebraic equations using Gauss-Seidel method:

$$
\begin{array}{ccccccc}
3x & - & 0.1y & - & 0.2z & = & 7.85 \\
0.1x & + & 7y & - & 0.3z & = & -19.3 \\
0.3x & - & 0.2y & + & 10z & = & 71.4
\end{array}
$$

using

$$
\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}
$$

as the initial guess and conduct three iterations

(20 marks)

QUESTION FOUR (20 MARKS)

a) Give the step by step procedure of direct power method in a format suitable for computer programming (8 marks)

b) Solve for the largest (in magnitude) eigenvalue of matrix A and the corresponding eigenvector x by the direct power method with $x^{(0)T} = \begin{bmatrix} 1.0 & 1.0 & 1.0 \end{bmatrix}$, using four iterations:

$$
A = \begin{bmatrix} 2 & -1 & 2 \\ 5 & -3 & 3 \\ -1 & 0 & -2 \end{bmatrix}
$$

(12 marks)

QUESTION FIVE (20 MARKS)

a) Use the contiuous Fourier series to approximate the square or rectangular wave function:

$$
f(t) = \begin{cases} -1 & -T/2 < t < T/2 \\ 1 & T/2 < t < T/4 \\ -1 & T/4 < t < T/2 \end{cases}
$$

(10 marks)

b) Given $y = 1.7 + \cos(4.189t + 1.0472)$. Generate 10 discrete values for the function at intervals of $\Delta t = 0.15$ for the range $t = 0$ to $t = 1.35$. Use this information to evatuate the coefficient of $y = A_0 + A_1 \cos(\omega_0 t) + B_1 \sin(\omega_0 t) + e.$ (10 marks)