



**JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY**

**SCHOOL OF MATHEMATICS AND ACTUARIAL SCIENCE**

**UNIVERSITY EXAMINATION FOR DEGREE OF BACHELOR OF SCIENCE**

**ACTUARIAL**

**3<sup>RD</sup> YEAR 2<sup>ND</sup> SEMESTER 2016/2017 ACADEMIC YEAR**

**REGULAR (MAIN)**

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**COURSE CODE: SAC 304**

**COURSE TITLE: ACTUARIAL LIFE CONTINGENCIES 1**

**EXAM VENUE:**

**STREAM: (BSc. Actuarial)**

**DATE:**

**EXAM SESSION:**

**TIME: 2.00 HOURS**

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**Instructions:**

- 1. Answer question 1 (Compulsory) and ANY other 2 questions**
- 2. Candidates are advised not to write on the question paper.**
- 3. Candidates must hand in their answer booklets to the invigilator while in the examination room.**

- **QUESTION 1**

- 1.(i) Define  ${}_tP_{xy}^-$  and show that  ${}_tP_{xy}^- = {}_tP_x + {}_tP_y - {}_tP_{xy}$  [3 marks]

- (ii) Hence or otherwise show that  $\ddot{a}_{xy} = \ddot{a}_x + \ddot{a}_y - \ddot{a}_{xy}$  [3 marks]

- 2) A last survivor policy provided \$ 10,000 payable immediately on the death of the second to die of a man aged 65 and his wife aged 60. Premiums are payable monthly in advance so long as at least one of the couple survived. The office which issued the policy used the following basis:

- Mortality : PMA92C20 and PFA92C20 ultimate

- Interest : 4 % pa

- Expenses : 2.5 % of all office premiums

- Find the monthly premium [5 marks]

- 3) A population is subject to 2 modes of decrement  $\alpha$  and  $\beta$ . For  $30 \leq x \leq 32$ . It is known that

- $\mu_x^\alpha = 0.05$  and  $\mu_x^\beta = (35 - x)^{-1}$ . For an individual in this population at exact age 30, calculate

- The probability that he will still be in the population at age 32 [3 marks]

- The probability that he will leave by mode  $\alpha$  before at age 32 [3 marks]

- 4. Given that  ${}_n p_x = 0.3$ ,  ${}_n p_y = 0.4$  and  ${}_n p_z = 0.6$ . Find the probability that of lives (x), (y) and (z) [3 marks]

- i. None will be alive in n years

- ii. Exactly one will survive n years

- iii. At least one will survive n years

- 3. An office issues a policy on the lives of a woman aged 60 and her husband aged 64. Under the policy, level premiums are payable annually in advance for 20 years or until the first death of the couple, if earlier

- On the death of the first couple, the survivor will receive an annuity of \$10,000 per annum payable weekly, beginning immediately on the first. Calculate the annual premium if the office uses the basis given below: [6 marks]

- Mortality : males PMA92C20

- Females PFA92C20

- Expenses : 20% of the first premium

- 5% of each premium after the first

- Interest : 4 % per annum

- **QUESTION 2**

- 1. The table below shows the independent rates of ill health retirement, withdrawals and death for a pension scheme for ages 20 and 40. Calculate the appropriate values for dependent rates of decrement at these ages. [4 marks]

Age	Ill-health	Withdrawal	Death
20	-	0.250	0.002
40	0.010	0.050	0.003

- 2. A life insurance company uses the three state illness-death model to calculate premium for a 2 year sickness policy issued to a healthy policyholder aged 58.  $S_t$  denotes the state occupied at age 58+t, so that  $S_0 = H$  (healthy) and  $S_t = H, S$  or  $D$  (healthy, sick or dead) for  $t=1,2$ .

- The transition intensities used by the insurer are defined in the following way :

- $p_{58+t}^{jk} = P(S_{t+1} = k / S_t = j), t=0,1$

- For  $t=0,1$  it is assumed that :

- $p_{58+t}^{HH} = 0.92$                        $p_{58+t}^{HS} = 0.05$                        $p_{58+t}^{HD} = 0.03$

- $p_{58+t}^{SH} = 0.65$                        $p_{58+t}^{SS} = 0.25$                        $p_{58+t}^{SD} = 0.10$

- The policy provides a benefit of \$5,000 at the end of each year if the policy is then sick and a benefit of \$20,000 at the end of the year of death. Calculate the expected present value of the benefits under this policy, assuming an interest of 5 % pa [8 marks]

- 3. A life office is proposing to issue 3 –year sickness benefit policies to lives aged 30. The benefit are \$50 per week during sickness within the next 3 years. There is no waiting period and the off period is as in the tables provided. Find the single premium on each of the following bases [8 marks]

- mortality: English life table No.12 –males
- Interest: i) 4% and ii) 5%
- Sickness: Manchester unity 1893-97 (AHJ)
- Expenses: none

**QUESTION 3**

- 1.i) Define in words  $1000 A_{x:y}^{\overline{2}}$  [1 marks]

- ii) Calculate: [8 marks]

- a)  $1000 \bar{A}_{30:40}^{-2}$
- b) The annual premium payable continuously until the second death for the above assurance in (a) with sum assured of \$1000.
- Basis:
- $\mu = 0.02$  for a life aged 30 exact at entry level throughout their life
- $\mu = 0.03$  for a life aged 40 exact at entry level throughout their life
- $\delta = 0.05$  throughout
- Expenses : nil
- iii.) Outline the main deficiency of the above premium paying scheme and suggest an alternative
  - [2 marks]
- 2. A multiple decrement table is subject to two forces of decrement  $\alpha$  and  $\beta$ . Under the assumption of uniform distribution of the independent decrements over each age.  $(aq)_x^\alpha = 0.15$  and  $(aq)_x^\beta = 0.07$ . Calculate  $q_x^\beta$  and  $q_x^\alpha$  [5 marks]
- 3. Define the following expressions in words and give an expression for each in terms of integral
  - [4 marks]
- a)  ${}_x q_{xy}^1$
- b)  $\bar{A}_{xy}^{-2}$

• **QUESTION 4 [20 marks]**

- 1. A population is subject to 2 modes of decrement  $\alpha$  and  $\beta$ . In the single decrement tables ;
- ${}_t p_{60}^\alpha = \frac{40-t}{40}$  for  $0 < t \leq 40$
- And
- ${}_t p_{60}^\beta = \left( \frac{40-t}{40} \right)^2$  for  $0 < t \leq 40$
- Calculate the value of  $(aq)_{60}^\alpha$  [7 marks]

- 2.i. Express  $A_{xx}^2$  in terms of  $a_x$  and  $a_{xx}$  and the rate of interest [3 marks]
- ii. Smith and Jones are both aged 60 years. A life office has been asked to issue a special joint life policy providing \$ 10,000 at the end of the year of death of the first to die of these two lives. In addition, if Smith is the second to die a further \$5,000 will be payable at the end of the year of his death. The policy is said to have annual premiums payable during the joint life of Smith and Jones
- Calculate the annual premium on the following basis [5 marks]
  - AM92 ultimate mortality
  - 4% interest
  - Expenses: 5% of all premiums with an additional initial expense of \$ 100
- iii. Write down (but do not EVALUATE) formulae for the reserve at duration 10 years (immediately before payment of the premium then due) on the premium basis, if
  - a) Both Smith and Jones are alive [2 marks]
  - b) John has died but Smith is alive [3 marks]

• **QUESTION 5 [18 marks]**

- 1. Consider each of the symbols listed below :
  - (a)  ${}_{10}q_{xy}^2$
  - (b)  $p_{xy}$
  - (c)  $\bar{A}_{xy}$
  - (d)  $\bar{A}_{\overline{xy}}$
  - (e)  $\bar{a}_{y/x}$
- Explain carefully the meaning of each of these symbols and calculate the value of each assuming that:
  - (x) is subject to a constant force of mortality of 0.01 pa
  - (y) is subject to a constant force of mortality of 0.02 pa
  - The lives are independent with respect to mortality

- The force of interest is 0.04 pa [14 marks]
- 2) A husband and wife, aged 70 and 65 respectively, effect a policy under which the benefits are :
- A lump sum of \$10,000 payable immediately on the first death and,
- (2) A reversionary annuity of \$ 5,000 payable continuously throughout the lifetime of the surviving spouse after the death of the first .Level premiums are payable in advance until the first death.
- Calculate the annual premium on the undernoted basis [6 marks]
- .Male's mortality : PMA92C20
- .Females mortality : PMA92C20
- Interest : 4% p.a
- Expenses : 10% of all premiums
- .Ignore the possibility of divorce