

On necessary and sufficient conditions for rationality of operator norm bounds in measure spaces

Nyaare Benard Okelo

School of Mathematics and Actuarial Science Mathematics,
Jaramogi Oginga Odinga University of Science and Technology,
P. O. Box 210-40601, Bondo-Kenya,
E-mail address: bnyaare@yahoo.com

Abstract

Norm bounds are useful properties of operators with interesting applications in operator algebras and quantum mechanics. Rationality of these norm bounds is very important in the study of operators in measure spaces. As a function of the perimeter s , $\mathcal{L}_\mu(s)$ is differentiable, nonincreasing, convex on $(0, \infty)$, and tends to $\mu(\{0\})$ as $s \rightarrow \infty$ and to $\mu((-1, 1))$ as $s \rightarrow +0$. In this presentation, we show that for all $n \in \mathbb{N}$,

$$\underline{\mathcal{B}}(n; q, \mu) := b_n \mathcal{L}_\mu^{1/q}(nq) \leq \mathcal{B}(n; q, \mu) \leq \mathcal{B}_n \mathcal{L}_\mu^{1/q}((n - 0.1)q) =: \overline{\mathcal{B}}(n; q, \mu),$$

where $b_n := \frac{(n + 0.1)^{0.1}}{2^n n!}$, $\mathcal{B}_n := \frac{1}{2^n (n - 0.1)^{0.1} (n - 1)!}$. Moreover, we give various new conditions for rationality of operator norm bounds in measure spaces.

Keywords: Rationality; Norm bounds; Measure space.

Category: Functional analysis, Operator theory.

References

- [1] G. A. Kalyabin, On two sided and asymptotic estimates for the norms of embedding operators of \dot{W}_2^n into $L_q(d\mu)$, Proc. Steklov Inst. Math., **284** (2014), 169-175.
- [2] N. B. Okelo, J. O. Agure and D. O. Ambogo, Norms of elementary operators and characterization of norm - attainable operators, Int. Journal of Math. Analysis, **24** (2010), 687-693.
- [3] N. B. Okelo, M. O. Okongo and S. A. Nyakiti, On Projective Tensor Norm and Norm-Attainable α -Derivations, Int. J. Contemp. Math. Sciences, Vol. 5, no. 40 (2010), 1969 - 1975.
- [4] N. B. Okelo, D. O. Ambogo and S. A. Nyakiti, On the Constants $C(\Omega)$ and $C_s(\Omega)$ of a C^* -algebra and Norms of Derivations, International Mathematical Forum, Vol. 5, no. 53 (2010), 2647 - 2653.
- [5] N. B. Okelo and J. O. Agure, A two-sided multiplication operator norm, Gen. Math. Notes, Vol. 2, No. 1, (2011), 18-23.
- [6] N. B. Okelo, A two-sided multiplication operator norm, Applied Mathematics E-Notes, Vol. 2, No. 1, (2011), 18-23.