JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY

SCHOOL OF MATHEMATICS AND ACTUARIAL SCIENCE

UNIVERSITY EXAMINATION FOR DIPLOMA IN LINUX ENGINEERING FOR IT PROFESSIONAL

1ST YEAR 1ST SEMESTER 2015/2016 ACADEMIC YEAR

KISUMU LEARNING CENTRE

COURSE CODE: SMA 2111

COURSE TITLE: DIFFERENTIAL AND INTERGRAL CALCULUS

EXAM VENUE: **STREAM: Diploma in Linux Engineering for It Professional**

DATE: 19/04/16   **EXAM SESSION: 9.00 – 10.30 AM**

TIME: 1.30 HOURS

<u>**Instructions:**</u>

1. **Answer question 1 (Compulsory) and ANY other 2 questions**

2. **Candidates are advised not to write on the question paper.**

3. **Candidates must hand in their answer booklets to the invigilator while in the examination room.**

QUESTION ONE

a) Consider the following function;

$$f(x) = \frac{16 - x^2}{4 + x},$$

where the domain of x is the set of all real numbers except -4. Does the function has a limit as x tends to a, where a is defined by -4. If it has, what is the limit? (2mks)

b) Define function and describe different types of functions (4mks)

c) Let f be the function from a set {a, b, c} to another set {1, 2, 3}, such that $f(a) = 2$, $f(b) = 3$ and $f(c) = 1$. Is f invertible function, and if it's, what is it's inverse (represent the results diagrammatically) (5mks)

d) Without using the rules for differentiation, apply the definition of the derivative to differentiate the following function.

$$f(x) = x^3 \qquad \text{(3mks)}$$

e) Consider a moving object with the velocity defined using the function,

$$v(t) = -t^2 + 5t.$$

Find the finite integral of $v(t)$ with respect to t, if t is defined in the closed interval [0, 6], t is the time (3mks)

f) Using quotient rule of differentiation, find the derivative of the function;

$$f(x) = \frac{x^2 - 1}{x^2 + 1}, \text{ (step by step workings must be shown)} \quad \text{(4mks)}$$

g) Define continuity of a function (2mks)

h) Determine whether the following piecewise function is continuous at 1

$$h(x) = \begin{cases} \frac{x^3 - 1}{x - 1} \\ 3 \end{cases} \qquad \text{for the first equation } x \text{ is not equal to 1 and } x \text{ is equal to 1 in}$$

the second equation. (3mks)

i) If f is the function with the constant value c, then

$$\frac{\partial f}{\partial x} = \frac{\partial c}{\partial x} = 0. \text{ Prove this} \qquad \text{(4mks)}$$

QUESTION TWO

a) Find the area below $f(x) = -x^2 + 4x + 3$ and above $g(x) = -x^3 + 7x^2 - 10x + 5$, over the interval $1 <= x <= 2$. (5mks)

b) Let f be the function given by $f(x) = x^2$ and g be the function given by $g(x) = x + 3$.

    i. Find the composition function of g and f, f and g

    ii. Is composition of g and f and f and g the same? Explain (5mks)

c) Let f_1 and f_2 be the function from R to R such that $f_1(x) = x^2$ and $f_2(x) = x - x^2$. What is the function $h(x)$ defined by $f_1 + f_2$ (4mks)

d) Let f be the function from the set of integers to the set of integers such that $f(x) = x + 1$.Is f function an invertible function? And if it is, what is the inverse (4mks)

e) Find the limit of function;

$$f(x) = 10x + 7 \text{ as } x \text{ tends to } 5 \quad (2\text{mks})$$

QUESTION THREE

a) State the squeeze theorem of the limit (3mks)

b) A function f is given by

$$f(x) = \frac{(x^2 + 4)^{1/2} - 2}{x^2}, \text{ what is limit of this function as x tends to 0.}$$

(All steps must be shown with appropriate explanations) (5mks)

c) Find the limit

$$\lim_{x \to 0} \frac{10x - 3 \sin x}{x} \qquad (5\text{mks})$$

d) Given the function $y = \cos x$. Show that the derivative of y is

$$\frac{\partial y}{\partial x} = -\sin x \qquad (7\text{mks})$$

QUESTION FOUR

a) The function $y = 6x - 10$ is the composite function of two function $y = 2u$ and $u = (3x - 5)$.Find the derivative of this function y using chain rule (2mks)

b) A dynamic blast propels a heavy rock straight up with a launch velocity of 160ft/sec. It reaches a height of $s(t) = 160t - 16t^2$ feet after t seconds.

    i.   How high does the rock go

    ii.   What is the velocity and speed of the rock when it is 256 ft above the ground on the way up? on the way down?

    iii.   When does the rock hit the ground

    iv.   What is the acceleration of the rock at any time t during its flight (after the blast) (13mks)

c) Find derivative of $f(x) = (x^2 + 1)(x^3 + 3)$ (5mks)

QUESTION FIVE

a) The trigonometric function is defined by $y = \sin x$. Using the definition of derivatives, show that the differentiation of the function y is

$$\frac{\partial y}{\partial x} = \cos x \qquad \text{(5mks)}$$

b) Evaluate the three integrals;

$A = \int_0^3(-x^2 + 9)dx$, $B = \int_0^4(-x^2 + 9)dx$ and $C = \int_4^3 -(x^2 + 9)dx$ and show that $A = B + C$ (10mks)

j) Determine whether the following piecewise function is continuous at 1

$$g(x) = \begin{cases} \frac{x^3 - 1}{x-1} \\ 2 \end{cases}$$ for the first equation x is not equal to 1 and x is equal to 1 in

the second equation.   (5mks)