COURSE TITLE: **VECTOR ANALYSIS**

COURSE CODE: **SMA 202**; TIME: 2 HOURS

Answer question ONE and any other TWO questions.

QUESTION 1

(a) Find the directional derivative of $\phi = x^2yz + 4xz^2$ at (1,-2,-1) in the direction $2\mathbf{i} - \mathbf{j} - 2\mathbf{k}$. **[5mks]**

(b) Find the projection of vector $\mathbf{A} = \mathbf{i} - 2\mathbf{j} + \mathbf{k}$ on the vector $\mathbf{B} = 4\mathbf{i} - 4\mathbf{j} + 7\mathbf{k}$ **[4mks]**

(c) A particle moves along a curve whose parametric equations are $x = e^{-t}; y = 2cos3t$ and $z = 2sin3t$ where t is the time. Determine the magnitude of the acceleration at $t = 0$. **[4mks]**

(d) Find the angle θ between the vectors $\mathbf{v}$ = (2,1,-1) and $\mathbf{w}$= (3,-4,1) **[4mks]**

(e) Given that $\mathbf{F}(x, y) = y\mathbf{i} + x^2\mathbf{j}$, evaluate the line integral

$$\int_C \mathbf{F}\, d\mathbf{r}$$

for the following parabolic curve;

$C_1 : r_1(t) = (4-t)i + (4t-t^2)j,\ 0 \le t \le 3$ **[6mks]**

(f) Find the volume of the parallelepiped with adjacent sides $\mathbf{u} = (2, 1, 3)$, $\mathbf{v} = (-1, 3, 2)$, and $\mathbf{w} = (1, 1, -2)$ **[4mks]**

(g) If $\mathbf{A} = (3x^2 + 6y)\mathbf{i} - 14yz\mathbf{j} + 20xz^2\mathbf{k}$, evaluate; $\int \mathbf{A} d\mathbf{r}$ from $(0, 0, 0)$ to $(1, 1, 1)$ along the following path; $x = t, y = t^2, z = t^3$ **[3mks]**

QUESTION 2

(a) A particle moves so that its position vector is given by $\mathbf{r} = cos\omega t\mathbf{i} + sin\omega t\mathbf{j}$ where ω is a constant. Show that

(i) the velocity $\mathbf{v}$ of the particle is perpendicular to $\mathbf{r}$ **[4mks]**

(ii) the acceleration $\mathbf{a}$ is directed towards the origin and has magnitude proportional to the distance from the origin **[3mks]**

(iii)$\mathbf{r} \times \mathbf{v}$ is a constant vector **[4mks]**

(b) (i) Find the unit tangent on the curve $x = t^2 + 1, y = 4t - 3, z = 2t^2 - 6t$ **[6mks]**

(ii) Determine the unit tangent vector at a point where $t = 2$ **[3mks]**

QUESTION 3

(a) Prove that $\nabla.(\nabla\phi) = 0$ **[4mks]**

(b) If $\mathbf{r}_1 = 2\mathbf{i} - \mathbf{j} + \mathbf{k}$, $\mathbf{r}_2 = \mathbf{i} + 3\mathbf{j} - 2\mathbf{k}$, $\mathbf{r}_3 = -2\mathbf{i} + \mathbf{j} - 3\mathbf{k}$ and $\mathbf{r}_4 = 3\mathbf{i} + 2\mathbf{j} + 5\mathbf{k}$, find scalars a, b, c such that: $\mathbf{r}_4 = a\mathbf{r}_1 + b\mathbf{r}_2 + c\mathbf{k}_3$. **[4mks]**

(c) Find the constants a, b, and c such that;

$\mathbf{v} = (x + 2y + az)\mathbf{i} + (bx - 3y - z)\mathbf{j} - (4x + cy + 2z)\mathbf{k}$ is irrotational. **[6mks]**

(d) If $\mathbf{v} = \mathbf{w} \times \mathbf{r}$, prove that $\mathbf{w} = \frac{1}{2}Curl\mathbf{v}$ where $\mathbf{w}$ is a constant vector. **[6mks]**

QUESTION 4

(a) Given that in the xy-plane, a force $F = x^2yi + xy^2j$ acts on a body as it moves between (0, 0) and (1, 1)

Determine the work done when the path is

(i) along the line $y = x$ **[3mks]**

(ii) along the curve $y = x^n$ **[3mks]**

(iii) along the x-axis to the point $(1, 0)$ and then along the line $x = 1$ **[4mks]**

(b) Verify Stokes theorem for $\mathbf{A} = (2x - y)\mathbf{i} - yz^2\mathbf{j} - y^2z\mathbf{k}$

where S is the upper half surface of the sphere $x^2 + y^2 + z^2 = 1$ and C is its boundary **[10mks]**

QUESTION 5

(a) Show that $\overrightarrow{F} = (2xy + z^3)\mathbf{i} + x^2\mathbf{j} + 3xz^2\mathbf{k}$ is a conservative force field. Find the work done in moving an object in this field from (1, -2, 1) to (3, 1, 5) **[10mks]**

(b) Given that $\overrightarrow{F} = 2xz\mathbf{i} - x\mathbf{j} + y^2\mathbf{k}$, evaluate

$\int \overrightarrow{F} dv$, where v is the region bounded by the surfaces $x = 0, x = 2$, $y = 0, y = 6, z = x^2, z = 4$ **[10mks]**