JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY

SCHOOL OF MATHEMATICAL & ACTUARIAL SCIENCE

UNIVERSITY EXAMINATION FOR THE BACHELORS DEGREE

4TH YEAR 1ST SEMESTER 2013/2014 ACADEMIC YEAR

CENTRE: MAIN SCHOOL BASED

COURSE CODE: SMA 405

COURSE TITLE: PARTIAL DIFFERENTIAL EQUATION 1

EXAM VENUE: CR 1 **STREAM: (BSc. Actuarial, Bed,B Sc)**

DATE: 29/4/2014 **EXAM SESSION: 9.00 – 11.00 AM**

TIME: 2 HOURS

Instructions:

1. **Answer question 1 (Compulsory)and ANY other 2 questions**
2. **Candidates are advised not to write on the question paper.**
3. **Candidates must hand in their answer booklets to the invigilator while in the examination room.**

Question 1[30 marks] COMPULSORY

(a) Consider the second order linear partial differential equation

$$a\frac{\partial^2 u}{\partial x^2}+b\frac{\partial^2 u}{\partial x\partial y}+c\frac{\partial^2 u}{\partial y^2}+d\frac{\partial u}{\partial x}+e\frac{\partial u}{\partial y}+fu+g=0, \quad :u(x,y)$$

where a, b, c, d, e, f, g are in general variable coefficients which may depend on real x or y with $u(x,y)$ as the dependent variable. Use discriminant $\Delta(a,b,c)$ theory to categorize; elliptic, parabolic and hyperbolic partial differential equations;

(i) $\frac{\partial^2 u}{\partial x^2}+4x^2y^{14}\frac{\partial^2 u}{\partial y^2}=11$ (ii) $\frac{\partial u}{\partial t}=121t^6\frac{\partial^2 u}{\partial x^2}$

(iii) $\frac{\partial^2 u}{\partial x^2}+23x^3\frac{\partial^2 u}{\partial y^2}=0$ (iv) $\frac{\partial^2 u}{\partial t^2}-t^2x^{12}\frac{\partial^2 u}{\partial x^2}=110t$. [10 marks]

(b) Given the partial differential equation

(i) $x\frac{\partial F}{\partial x}-2y\frac{\partial F}{\partial y}=31xy^2$ (ii) $x^2y\frac{\partial^2 F}{\partial x^2}-y^2\frac{\partial^2 F}{\partial y^2}+x\frac{\partial F}{\partial x}-2y\frac{\partial F}{\partial x}\frac{\partial F}{\partial y}=0$

(iii) $x^2\frac{\partial^{13} F}{\partial x^{13}}-y^2\left(F\frac{\partial^2 F}{\partial y^2}\right)^4+x\frac{\partial F}{\partial x}-y\frac{\partial F}{\partial y}=0$

State in each case, the order, degree and whether linear or nonlinear. [6marks]

(c) Use characteristic method to solve the linear partial differential equation
$u_x + u_y = 2$
subject to the initial condition $u(x, 0) = x^2$. [8 marks]

(d) Determine the function $z(x,y)$ which satisfies the linear second order partial differential equation $\left(D^2-DD'-6D'^2\right)z=0$. [6marks]

Question 2 [20marks]

Given the function $F(x,y)=13y^2+104x^2+26x^4+14000-52x^2y$

(i) Find $\frac{\partial F}{\partial x}$, $\frac{\partial F}{\partial y}$, [4 marks]

(ii) Find $\frac{\partial^2 F}{\partial x^2}$, $\frac{\partial^2 F}{\partial y^2}$ and $\frac{\partial^2 F}{\partial x \partial y}$ [5marks]

(iii) Determine and distinguish all the stationary points of F [11 marks]

Question 3[20marks]

(a) Eliminate the arbitrary functions f , g from the equation

$$u = f\left(x+y\right)+g\left(x-y\right)+\frac{1}{4}x\left(x-y\right)^2$$ [6marks]

(b) Solve the linear second order partial differential equation

$$\left(4D^2-12DD'+9D'^2\right)u=0$$ [6marks]

(c) Solve the linear second order partial differential equation

$$\left(D^3-3D^2D'-4D'^3\right)u=e^{5x+7y}$$ [8marks]

Question 4 [20marks]

(a) Solve the equation
$-yu_x + xu_y = u$
subject to the initial condition
$u(x, 0) = \psi(x)$. [10 marks]

(b) Eliminate the arbitrary functions f , g , h from the equation

(i) $u = f\left(x-at+iby\right)+g\left(x-at-iby\right) \quad : i=\sqrt{-1}$

(ii) $u = f\left(x+y\right)+g\left(x-y\right)+h\left(2x+y\right)-\frac{1}{2}x\left(x-y\right)^2 e^{x+y}$ [10 marks]

Question5 [20marks]

(a)Solve the initial boundary value heat equation

$u_t = \frac{1}{100}u_{xx}, \quad 0<x<1, t>0$

satisfying the conditions

$u\left(0,t\right)=10,\ u\left(1,t\right)=10 \quad 0<x<1, t>0,\ u\left(x,0\right)=1+\sin 2® x, \quad 0<x<1$ [13 marks]

(b)Determine the critical points of the curve $\Phi\left(x,y\right)=x^3+y^2-3\left(x+y\right)+1100$ [7marks]

LAPLACE TRANSFORMS TABLE

$J_0(t)$	$\frac{1}{\sqrt{s^2+1}}$
$\sin bt$	$\frac{b}{s^2+b^2}$
$e^{-at}\sin bt$	$\frac{b}{(s+a)^2+b^2}$
$e^{-at}\cos bt$	$\frac{(s+a)}{(s+a)^2+b^2}$
$e^{-at}t^n$	$\frac{\Gamma(n+1)}{(s+a)^{n+1}}$ $n>-1$
t^n	$\frac{n!}{s^{n+1}}$
$e^{-at}t^n$	$\frac{n!}{(s+a)^{n+1}}$
$\frac{dy}{dt}$	$sY-y_0$ $Y=L(y)$
$\frac{d^2y}{dt^2}$	$s^2Y-sy_0-y_0'$ $Y=L(y)$
$\frac{\partial u(x,t)}{\partial t};s$	$sU(x,s)-u(x,0)$
$\frac{\partial^2 u(x,t)}{\partial t^2};s$	$s^2U(x,s)-su(x,0)-u_t(x,0)$
$\frac{\partial u(x,t)}{\partial x};s$	$\frac{dU(x,s)}{dx}$
$\frac{\partial^2 u(x,t)}{\partial x^2};s$	$\frac{d^2U(x,s)}{dx^2}$

$\frac{\partial^2 u(x,t)}{\partial x \partial t}; s$	$s\frac{dU(x,s)}{dx} - \frac{du(x,0)}{dx}$
$J_0(t)$ is the Bessel function of order zero. $L^{-1}\left\{\text{¤}(s)\right\} = e^{-at}L^{-}\left\{\text{¤}(s-a)\right\}$, $L\left\{e^{-at}f(t)\right\} = L\left\{f(t)\right\}_{s\to s+a}$	

LAPLACE TRANSFORMS