JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY

SCHOOL OF MATHEMATICAL & ACTUARIAL SCIENCE

UNIVERSITY EXAMINATION FOR THE MASTERS DEGREE

IN APPLIED MATHEMATICS

1ST YEAR 1ST SEMESTER 2013/2014 ACADEMIC YEAR

KISUMU LEARNING CENTRE

COURSE CODE: SMA 842

COURSE TITLE: NUMERICAL ANALYSIS II

EXAM VENUE: **STREAM: (MSc.)**

DATE: 02/05/2014 **EXAM SESSION: 9.00 – 12.00 NOON**

TIME: 3 HOURS

Instructions:

1. **Answer question 1 (Compulsory)and ANY other 2 questions**
2. **Candidates are advised not to write on the question paper.**
3. **Candidates must hand in their answer booklets to the invigilator while in the examination room.**

Question 1 [20 marks]

a) Determine the **Padé** rational approximation $r_{2,3}$ to $f(x)=e^{-x}$ of degree 5 with n=2,and m=3. Take the Maclaurin expansion for $f(x)=e^{-x}$ as

$; P_5(x)=\left(1-x+\frac{x^2}{2}-\frac{x^3}{6}+\frac{x^4}{24}-\frac{x^5}{120}..\right)$ [12 marks]

b) Compute the error $E_{m,n}(x)=\left|e^{-x}-r(x)\right|$ in the Padé's rational approximations $r_{m,n}(x)$ to $f(x)=e^{-x}$ at points $x=0, \pm0.2, \pm0.4, \pm0.6, \pm0.8, \pm1.0$. On same table display the values of x , e^{-x} , $r(x)$, $\left|e^{-x}-r(x)\right|$.Comment appropriately on the error propagation [6 marks]

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Question 2 [20 Marks]

(a) Derive the Trapezoidal method $y_{n+1}=y_n+\frac{h}{2}\left[f(x_n,y_n)+f(x_{n+1},y_{n+1})\right]; h=x_{n+1}-x_n$ step size to approximate the first order initial value problem

$y'=f(x,y),\ a\le x\le b,\quad y(a)=\sim$ [8 marks]

(b) Use the Euler's method , with step size $h=0.02$, to estimate $y(0.2)$ where y satisfies ordinary first order differential equation,

$y'=\frac{y-y^2}{1+x^2}\ ;\ y(0)=10$

On the same table display the results; n, x_n, y_n [5 marks]

Question3 [20 marks]

(a) For the second order boundary- value problem

$y''+\frac{4}{x}y'-\frac{16}{x^2}y=\frac{\sin(\ln x)}{x^2},\ 1\le x\le 2\quad ;\quad y(1)=1,\ y(2)=2,$

show that it has a unique solution $y(x)$ over the sub domain D where

$D=\{(x,y,y')|\ ,\ 1\le x\le 2, -\infty<y<\infty, -\infty<y'<\infty\}$ [8 marks]

(b) (i) Construct a finite difference scheme to the second order boundary- value problem

$y''+\frac{4}{x}y'-\frac{16}{x^2}y=\frac{\sin(\ln x)}{x^2},\ 1\le x\le 2\quad ;\quad y(1)=1,\ y(2)=2,$

(ii) Use the above numerical scheme with step-size $h=0.2$ and an appropriate iterative method to approximate the solution of the above given boundary- value problem. [12 marks]

Question4 [20 marks]

Approximate the solution of the nonlinear initial boundary value problem

$y'' = y^2 - 2yy'; \quad y'(0) = -1, \; y(1) = 0.5$.

Use step size $h = \Delta x = x_{i+1} - x_i = 0.25$.

Question5 [20 marks]

Given the system of linear equations $M\,\underline{X} = \underline{b}$ where $M = \begin{bmatrix} 2 & -1 & 0 & 0 \\ -1 & 2 & -1 & 0 \\ 0 & -1 & 2 & -1 \\ 0 & 0 & -1 & 2 \end{bmatrix}$,

Prove that Gauss Seidel iterative scheme for the linear system will converge much faster than Gauss Jacobi's iterative scheme. [20 marks]