JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY

SCHOOL OF MATHEMATICAL & ACTUARIAL SCIENCE

UNIVERSITY EXAMINATION FOR THE MASTERS DEGREE

IN APPLIED MATHEMATICS

1^{ST} YEAR 1^{ST} SEMESTER 2013/2014 ACADEMIC YEAR

KISUMU LEARNING CENTRE SCHOOL BASED

COURSE CODE: SMA 849

COURSE TITLE: NUMERICAL SOLUTION OF PDE

EXAM VENUE: **STREAM: (BSc. Actuarial, Bed,)**

DATE: 29/4/2014 **EXAM SESSION: 9.00 – 12.00 NOON**

TIME: 3 HOURS

Instructions:

1. **Answer question 1 (Compulsory)and ANY other 3 questions**
2. **Candidates are advised not to write on the question paper.**
3. **Candidates must hand in their answer booklets to the invigilator while in the examination room.**

Question 1

(a) Construct Crank-Nicolson implicit finite-difference scheme as applied to the second order linear parabolic partial differential equation

$$\frac{\partial u}{\partial t} = \;{}^{2}\frac{\partial^2 u}{\partial x^2}, \qquad a \le x \le b, \qquad t > 0$$

subject to $u(0,t) = \sim$, $u(1,t) =$ ¤ $\quad t > 0$

and $u(x,0) = \sin x \quad a \le x \le b$. [10 marks]

(b) Determine the stability condition for the Crank-Nicolson implicit finite-difference scheme in part (a) above [10 marks]

Question 2

(a) Use a discriminant Δ theory to categorize ; elliptic, parabolic and hyperbolic partial differential equations given below.

(i) $x^2\frac{\partial^2 u}{\partial x^2} + 440\frac{\partial^2 u}{\partial y^2} = x$ (ii) $10\frac{\partial u}{\partial t} = -3\frac{\partial^2 u}{\partial x^2}$ (iii) $\frac{\partial^2 u}{\partial x^2} + y\frac{\partial^2 u}{\partial y^2} = 10$ (iv) $\frac{\partial u}{\partial t} + t^{14}y^2\frac{\partial^2 u}{\partial t \partial x} = \frac{\partial^2 u}{\partial x^2}$ [10 marks]

(b) Let $f\left(x, u, u_1, u_2, u_3 ...., u_{m+1}\right) = 0$ be an $(m+1)^{th}$ order partial differential equation and $F(u_{ij}) = 0$ describe a finite difference scheme to it.

Describe the conditions under which the numerical scheme will be consistent and stable to the $(m+1)^{th}$ **order** partial differential equation. [6 marks]

(c) Give the necessary and sufficient for the numerical scheme to converge to the exact solution. [4 marks]

Question 3

(a) Construct an explicit finite-difference scheme as applied to the nonhomogenous parabolic equation

$$\frac{\partial u}{\partial t} = \left(\frac{1}{100}\right)\frac{\partial^2 u}{\partial x^2} + 12, \qquad 0 \le x \le 10, \qquad 0 < t$$

subject to $u(0,t) = u(1,t) = 0 \quad 0 < t$

and $u(x,0) = x(1-x) + \sin® x \quad 0 < x < 10$. [5 marks]

(b) Obtain a molecular formula for problem (a) above applied to the solution grid over region $W = \{(x,t) : 0 \le x \le 10, 0 \le t\}$ with; $h = \Delta x = 2$; $k = \Delta t = 0.001$.

State the stability condition for the molecular formula employed. And hence compute, the numerical solutions U_{ij} ; for the three time levels $j = 0, 1, 2$. [15 marks]

Question 4

Given the wave equation

$$\frac{\partial^2 u}{\partial t^2} = 40\frac{\partial^2 u}{\partial x^2}, \qquad 0 < x < 1, \quad t > 0$$

subject to boundary conditions $u(0,t) = u(1,t) = 0 \qquad \text{for } t > 0$

and initial conditions $u(x,0) = \sin(2® x),\ u_t(x,0) = 0,\ 0 < x < 1, \quad t = 0\ .\ h = \Delta x\ , t = 0$.

(a) Construct the explicit finite difference scheme to it. [8 marks]

(b) State the stability of the explicit finite difference scheme in part (i) above. [2 marks]

(c) Compute the approximations $U_{i,j}\ ;\ j = 1(\mathit{first-time\ level})\ ,\ i = 0,1,2,3,4,5.$ to the exact solutions $u(x_i, t_j)$ using $h = \Delta x = 0.2,\ k = \Delta t = 0.05\ h = \Delta x$. [10 marks]

Question 5

On the square $D = \{(x, y) : 0 \le x \le 2, 0 \le y \le 2\}$ consider the Dirichlet problem for the Poisson's equation,

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 10\ in\ D$$

$$u = g(x, y)\ on\ S$$

(a) Use finite difference method with equal mesh spacing $h = \Delta x = \Delta y = \frac{1}{2}$, defined on D to discretize the Dirichlet problem, assuming . $g(x, y) = 0\ on\ S$. [15 marks]

(b)Show that difference scheme takes the form $A\underline{U} = \underline{B}\ :\ A_{9\times 9} real\ ,\ symmetric\ matrix.$

Deduce that the numerically computed solution $\underline{U}$ is unique [5 marks]