JA

JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY

SCHOOL OF MATHEMATICS AND ACTUARIAL SCIENCE

UNIVERSITY EXAMINATION FOR DEGREE OF BACHELOR OF SCIENCE ACTUARIAL

4TH YEAR 1ST SEMESTER 2015/2016 ACADEMIC YEAR

MAIN CAMPUS - RESIT

COURSE CODE: SMA 405

COURSE TITLE PARTIAL DIFFERENTIAL EQUATION 1

EXAM VENUE: LAB 1 **STREAM: (BSc. Actuarial/BED)**

DATE: 04/5/16 **EXAM SESSION: 2.00 – 4.00 PM**

TIME: 2.00 HOURS

Instructions:

1. **Answer question 1 (Compulsory) and ANY other 2 questions**
2. **Candidates are advised not to write on the question paper.**
3. **Candidates must hand in their answer booklets to the invigilator while in the examination room.**

Show all the necessary working

Question 1[30 marks] Compulsory

(a) Given the partial differential equation

(i) $x\frac{\partial F}{\partial x}-y\frac{\partial F}{\partial y}=3xy^2$ (ii) $x^2\frac{\partial^2 F}{\partial x^2}-y^2\frac{\partial^2 F}{\partial y^2}+x\frac{\partial F}{\partial x}-y\frac{\partial F}{\partial x}\frac{\partial F}{\partial y}=0$

(iii) $x^2\frac{\partial^3 F}{\partial x^3}-y^2\left(\frac{\partial^2 F}{\partial y^2}\right)^4+x\frac{\partial F}{\partial x}-y\frac{\partial F}{\partial y}=0$

State in each case, the order, degree and whether linear or nonlinear. [9marks]

(b) Consider the second order linear partial differential equation

$\frac{\partial^2 u}{\partial x^2}+2\frac{\partial^2 u}{\partial x\partial y}+x\frac{\partial^2 u}{\partial y^2}=0$,

(i) Classify the partial differential equation

(ii) Obtain the characteristic equation of the partial differential equation

(i) Solve the partial differential equation. [10 marks]

(c) Consider the second order linear partial differential equation

$$a\frac{\partial^2 u}{\partial x^2}+b\frac{\partial^2 u}{\partial x\partial y}+c\frac{\partial^2 u}{\partial y^2}+d\frac{\partial u}{\partial x}+e\frac{\partial u}{\partial y}+fu+g=0, \quad :u(x,y)$$

where a, b, c, d, e, f, g are in general variable coefficients which may depend on real x or y with $u(x,y)$ as the dependent variable. Use discriminant $\Delta(a,b,c)$ theory to categorize; elliptic, parabolic and hyperbolic partial differential equations ;

(i) $\frac{\partial^2 u}{\partial x^2}+4\frac{\partial^2 u}{\partial y^2}=1$ (ii) $\frac{\partial u}{\partial t}=t^6\frac{\partial^2 u}{\partial x^2}$ (iii) $\frac{\partial^2 u}{\partial x^2}+3\frac{\partial^2 u}{\partial y^2}=0$ (iv) $\frac{\partial^2 u}{\partial t^2}-\frac{\partial^2 u}{\partial x^2}=0$. [10 marks]

(d) Solve the first order partial differential equation $\frac{\partial z}{\partial x}z-\frac{\partial z}{\partial y}z=z^2+(x+y)^2$ [5marks]

(e) Determine the function $z(x,y)$ which satisfies the linear second order partial differential

equation $\left(D^2+DD'-6D'^2\right)z=0$ [6marks]

Question 2 [20marks]

Given the function $F(x,y,z)=8x^2+24y^2+16z^2+24x+16z+1111$,

(i) Find $\frac{\partial F}{\partial x}$, $\frac{\partial F}{\partial y}$, [4 marks]

(ii) Find $\frac{\partial^2 F}{\partial x^2}$, $\frac{\partial^2 F}{\partial y^2}$ and $\frac{\partial^2 F}{\partial x \partial y}$ [5marks]

(iii) Determine and distinguish all the stationary points of F [11 marks]

Question 3[20marks]

(a) Eliminate the arbitrary functions f, g from the equation

$$u=f(x+y)+g(x-y)+\frac{1}{4}x(x-y)^2$$ [6marks]

(b) Solve the linear second order partial differential equations

(ii) $\left(4D^2-12DD'+9D'^2\right)z=0$ [6marks]

(iii) $\left(D^3-3D^2D'-4D'^3\right)u=e^{8x+2y}$ [8marks]

Question 4 [20marks]

(a) Use characteristic method to solve the linear partial differential equation
$u_x+u_y+u=1$, subject to the initial condition $u=\sin x$, on $y=x+x^2$, $x>0$.[14 marks]

(b) Eliminate the arbitrary functions f, g from the equation

$u = f\left(x - at + by\right) + g\left(x - at - by\right)$ **[6 marks]**

Question 5[20marks]

Solve the initial boundary value heat equation

$u_t = u_{xx}\,, \quad 0 < x < 1\,, t > 0$

satisfying the conditions

$u\left(0,t\right) = 100,\; u\left(1,t\right) = 100 \quad 0 < x < 1\,, t > 0$

$u\left(x,0\right) = 1 + \sin \pi x, \quad 0 < x < 1$ **[20 marks]**