



**JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY**

**SCHOOL OF MATHEMATICS AND ACTUARIAL SCIENCE**

**UNIVERSITY EXAMINATION FOR DEGREE OF BACHELOR OF SCIENCE**

**ACTUARIAL**

**4<sup>TH</sup> YEAR 1<sup>ST</sup> SEMESTER 2018/2019 ACADEMIC YEAR**

**REGULAR (MAIN)**

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**COURSE CODE: SAC 403**

**COURSE TITLE: ACTUARIAL LIFE CONTINGENCIES II**

**EXAM VENUE:**

**STREAM: (BSc. ACTUARIAL SCIENCE)**

**DATE:**

**EXAM SESSION:**

**TIME: 2.00 HOURS**

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**Instructions:**

- 1. Answer question 1 (Compulsory) and ANY other 2 questions**
- 2. Candidates are advised not to write on the question paper.**
- 3. Candidates must hand in their answer booklets to the invigilator while in the examination room.**

**QUESTION 1 [COMPULSORY] [30 Marks]**

(a) Explain the difference between a profit vector and a profit signature.

[4 Marks]

(b) Write down in the form of symbols, and also explain in words, the expressions “expected death strain” and “actual death strain”. [8 Marks]

(c) Consider an year of death  $n$ -year endowment assurance for a sum assured of 1 without profits, payable at the end of the year of death (if this occurs within the term of the policy). There are level premiums of  $P''$ , payable annually in advance for  $n$  years until earlier death of the policyholder, who was aged  $x$  at the issue date. The premium and reserving basis includes the following allowances for expenses: expenses of  $e$  on the payment of each premium, with additional initial expenses of  $I$  (so the total initial expense is  $I + e$ ).

Show that the “Zillmerised reserve” required at duration  $t$ , just before payment of the premium then due is given by

$${}_tV^Z = (1 + I){}_tV - I$$

where  ${}_tV$  denotes the corresponding net premium reserve, i.e

$$\begin{aligned} {}_tV &= {}_tV_{x:\overline{n}|} \\ &= 1 - \frac{\ddot{a}_{x=t:\overline{n-t}|}}{a_{x:\overline{n}|}} \end{aligned}$$

[6 Marks]

(d)(i) A ten year regular premium term assurance policy is issued a group of lives aged 40. The sum assured 20,000 and is payable at the end of

the year of death. Write down an expression for the retrospective reserve immediately before the 6th premium is due, assuming that reserves are calculated on the same basis as premium  $P$ . **[5 Marks]**

(ii) A temporary annuity of 3000 payable annually in arrears for a term of 10 years was purchased one year ago by a life then aged exactly 60 by payment of a single premium. Show algebraically that the current retrospective and current prospective net reserves are equal assuming that the pricing and reserving bases are the same. **[7 Marks]**

### QUESTION 2[20 MARKS]

(a) Show how you would modify the premium equation below to allow for expenses indicated.

$$P\ddot{a}_{x:\overline{n}|} = SA_{x:\overline{n}|}$$

(i) initial expense of 2% of sum assured **[3 Marks]**

(ii) renewal expenses of 2% of each premium, including yje first. **[3 Marks]**

(iii) claim expenses of 2% of sum assured. **[3 Marks]**

(iv) initial expenses of 50% of first premium plus renewal expenses 3% of each premium including the first. **[3 Marks]**

b) Consider a whole life policy with sum assured 1 without profits, payable immediately on the death of  $(x)$ . The policy was issued  $t$  years ago by level annual premiums payable continuously throughout life. Derive the formula for the net premium reserve  ${}_tV$  (on a given mortality and interest basis)

**[8 Marks]**

**QUESTION 3[20 MARKS]**

A life insurance company issues the following policies:

- 15-year term assurances with a sum assured of 150,000 where the death benefit is payable at the end of the year of death
- 15-year pure endowment assurances with a sum assured of 75,000
- 5-year single premium temporary immediate annuities with an annual benefit payable in arrear of 25,000

On 1 January 2002, the company sold 5,000 term assurance policies and 2,000 pure endowment policies to male lives aged 45 exact and 1,000 temporary immediate annuity policies to male lives aged 55 exact. For the term assurance and pure endowment policies, premiums are payable annually in advance. During the first two years, there were fifteen actual deaths from the term assurance policies written and five actual deaths from each of the other two types of policy written.

(a) Calculate the death strain at risk for each type of policy during 2004.

**[6 Marks]**

(b) During 2004, there were eight actual deaths from the term assurance policies written and one actual death from each of the other two types of policy written. Calculate the total mortality profit or loss to the office in the year 2004.

Basis: Interest- 4% per annum, Mortality- AM92 Ultimate for term assurances and pure endowments and PMA92C20 for annuities. **[14 Marks]**

**QUESTION 4[20 MARKS]**

A life insurance company issues a five-year without profits endowment assurances for an annual premium of 3600 and a sum assured of 20000 payable on maturity or at the end of the year of death if earlier.

The company uses the following assumptions for the profit testing:

Year	Mortality rate	Surrender rate	Expenses at the start of year per policy	Reserves at start of year per policy	Surrender value at the end of year per policy
1	0.01	0.05	750	3,100	2,800
2	0.01	0.05	15	6,800	6,250
3	0.01	0.05	15	10,900	10,000
4	0.01	0.05	15	15,300	14,500
5	0.01	0	15	-	-

Surrenders occur only at the end of the year immediately before a premium is paid. The surrender rates shown in the table above are applied to the number of policies in force at each year end.

(a) Set out the column headings and formulae you would use to calculate the profit arising each year per policy in force at the beginning of the year.

**[8 Marks]**

(b) Calculate (to the nearest 1%) the internal rate of return obtained by the company.

**[12 Marks]**

**QUESTION 5[20 MARKS]**

A special endowment policy pays a sum assured of 20,000 to a life who is currently aged exactly 57 after three years or at the end of the year of earlier death. Annual reversionary bonuses are declared at the end each policy year, and a terminal bonus is payable at maturity only. Policies may be surrendered only at the end of each policy year. On surrender, the policyholder receives a return of premiums with interest calculated at the rate of 3% per annum. A level premium of 8,000 is paid at the start of each year. The premium basis is as follows;

- Interest- 7% per annum
- Mortality- AM92 select
- Surrender rates -15% of all policy in force at the end of the year, 5% of all policy in force at the end of year 2.
- Reversionary bonuses -6% per annum compound
- Terminal bonuses-10% of all other benefits payable at maturity
- Expenses -Initial 500, Renewal 30 at the start of year 2, 35 at the start of year 3, Termination 100 per termination
- Reserves -Net premium reserves, using AM92 ultimate mortality and 4% per annum interest

Calculate the profit signature according to the premium basis.

**[20 Marks]**