JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY

SCHOOL OF MATHEMATICS AND ACTUARIAL SCIENCE

UNIVERSITY DRAFT EXAMINATION FOR MSC IN MATHEMATICS

1st YEAR 1st SEMESTER 2017/2018 ACADEMIC YEAR

KISUMU CAMPUS

COURSE CODE: SMA 840

COURSE TITLE: METHODS OF APPLIED MATHEMATICS I

EXAM VENUE: **STREAM: MSc Y1S1**

TIME: 3 HOURS **EXAM SESSION:**

Instructions:

Answer any three questions

1. **Show all the necessary working**
2. **Candidates are advised not to write on the question paper**
3. **Candidates must hand in their answer booklets to the invigilator while in the examination room**

Question 1 [20 marks]

(a) Find Laplace transform of (i) $\cosh t\delta(t-a)$ (ii) $e^{-40t}\sin 30t$ (ii) $\dfrac{\cos 6t-\cos 4t}{t}$ [11 marks]

(b) For the fourth order initial value ordinary differential equation

$$x^{iv}(t)+13x''(t)+36x(t)=0;\ x(0)=x''(0)=0,\ x'(0)=2,\ x'''(0)=-13$$

obtain $x(t)$. [9 marks]

Question 2 [20 marks]

(a) If $F(s)=\dfrac{s}{(s-6)(s^2+16)}$ use the Laplace transforms to find $f(t)$. [4 marks]

(b) Give matrix $A=\begin{bmatrix}-2 & 4\\ -1 & 3\end{bmatrix}$

(i) compute the exponential matrix e^{At}. [9 marks]

(ii) Prove that $\left[e^{At}\right]^{-1}=\left[e^{A(-t)}\right]$ [5 marks]

(iii) Deduce that $\left[e^{A(t=0)}\right]=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$ [2marks]

Question3 [20 marks]

(a) For the $L-R-C$ electrical network in figure1 below, show that the current $i_1(t)$ and charge $q(t)$ satisfies the system of differential equations

$$\frac{d}{dt}\begin{pmatrix}i_1\\ q\end{pmatrix}=\begin{pmatrix}\dfrac{R_1R_2}{L(R_1+R_2)} & \dfrac{R_1}{CL(R_1+R_2)}\\ \dfrac{R_1}{(R_1+R_2)} & \dfrac{1}{C(R_1+R_2)}\end{pmatrix}\begin{pmatrix}i_1\\ q\end{pmatrix}+\begin{pmatrix}\dfrac{E}{L}\\ 0\end{pmatrix}.$$

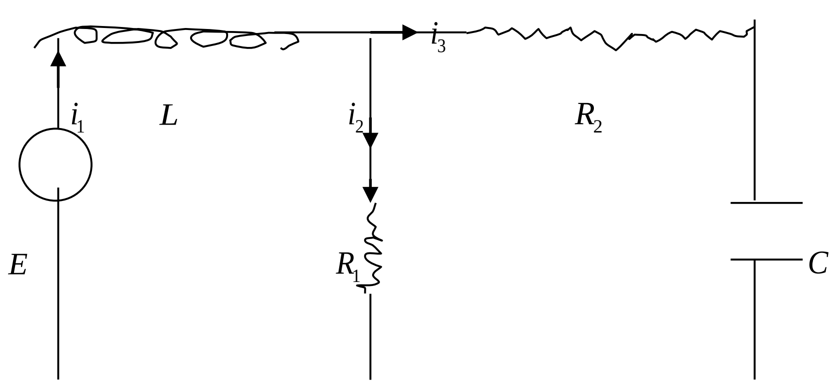

Fig 1

Use variation of parameters method or otherwise solve for system if
$R_1 = 50\Omega \ , R_2 = 00\Omega \ , L = 5h \ , C = 0.004 f \ \ and \ \ E(t) = 10V$ [13marks]

(b) Apply Fourier transform method to solve the Laplace boundary value problem
$u_{xx} + u_{yy} = 0, \quad -\infty < x < \infty \ , \ y > 0 \ ; \ u(x,0) \ = g(x)$ [7marks]

Question4 [20 marks].
(a) Solve the initial boundary value problem

$u_t = u_{xx} \, , \quad 0 < x < 1 \, , t > 0$ satisfying the conditions

$u(0,t) = 1, \ u(1,t) = 1 \quad 0 < x < 1 \, , t > 0$

$u(x,0) = 1 + \cos 2\pi x, \quad 0 < x < 1$ [17 marks]

(b) Determine the transient value of $u(x,t)$ [3 marks]

Question5 [20 marks]
Given the system of first order ordinary differential equations
$\frac{dx}{dt} = 5x + y + 3z + 5 \ , \quad \frac{dy}{dt} = x + 7y + z \quad , \frac{dz}{dt} = 3x + y + 5z$

(i) Express the system in the matrix form $\dot{\underline{X}} = A\underline{X} + \underline{f}(t)$

(ii) Show that $\underline{u} = [1,1,1]^t \ , \underline{v} = [-1,0,1]^t$ are eigenvectors of A

(iii) Determine $\lambda_1 \ , \lambda_2 , \lambda_3$, the eigenvalues of A and verify that $trace(A) = \lambda_1 \ , \lambda_2 , \lambda_3$

(iv) Determine $\Phi(t)$, the fundamental matrix of the system

(v) Obtain $\underline{X}$ the general solution of the system [20 marks]

LAPLACE TRANSFORMS TABLE

$f(t)$	Laplace transform of $f(t)$
1	$\frac{1}{s}$
t	$\frac{1}{s^2}$
e^{at}	$\frac{1}{s-a}$
$\cos bt$	$\frac{s}{s^2+b^2}$
$\sin bt$	$\frac{b}{s^2+b^2}$
$e^{-at}\sin bt$	$\frac{b}{(s+a)^2+b^2}$
$e^{-at}cos\,bt$	$\frac{(s+a)}{(s+a)^2+b^2}$
$e^{-at}t^n$	$\frac{\Gamma(n+1)}{(s+a)^{n+1}} \quad n>-1$
t^n	$\frac{n!}{s^{n+1}}$
$e^{-at}t^n$	$\frac{n!}{(s+a)^{n+1}}$
$\frac{dy}{dt}$	

$\frac{d^2y}{dt^2}$ $\frac{d^ny}{dt^n}$ u_t u_{tt} u_{x^m} u_{xt} $J_0(t)$	$sY - y_0 \quad ; \quad Y = L(y)$ $s^2Y - sy_0 - y_0' \;;\; Y = L(y)$ $s^nY - s^{n-1}y_0 - s^{n-2}y_0 - s^{n-3}y_0 - .... - sy^{(n-2)}(0) - y^{(n-1)}(0)$ $sU(x,s) - u(x,0); \quad U(x,s) = L\left[u(x,t)\right]$ $s^2U(x,s) - su(x,0) - u_t(x,0);$ $\frac{d^m}{dx^m}\left(U(x,s)\right)$ $s\frac{d}{dx}U(x,s) - \frac{d}{dx}u(x,0)$ $\frac{1}{\sqrt{s^2+1}}$
$t^n f(t)$ $\frac{f(t)}{t}$	$(-1)^n \frac{d^n\{F(s)\}}{ds^n}$ $\int_s^\infty F(s)\,ds$
$f^{(n)}(t)$ $\delta(t-a)$	$s^nF(s) - s^{n-1}f(0) - s^{n-2}f(0) - ....sf^{(n-2)}(0) - f^{(n-1)}(0)$ e^{-as}
$J_0(t)$	$\frac{1}{\sqrt{1+s^2}}$
$\frac{\partial u(x,t)}{\partial t}$	$sU(x,s) - u(x,0)$

$\frac{\partial u(x,t)}{\partial t}$	$sU(x,s)-u(x,0)$
$\frac{\partial^2 u(x,t)}{\partial t^2}$	$s^2U(x,s)-su(x,0)-u_t(x,0)$
$\frac{\partial u(x,t)}{\partial x}$	$\frac{dU(x,s)}{dx}$
$\frac{\partial^2 u(x,t)}{\partial x^2}$	$\frac{d^2U(x,s)}{dx^2}$
$\frac{\partial^2 u(x,t)}{\partial t\partial x}$	$s\frac{dU(x,s)}{dx}-\frac{du(x,0)}{dx}$
$L^{-1}[F(s)G(s)] = \int_0^t f(t-u)g(u)du$	
$H(t-a)$	$\frac{e^{-as}}{s}$
$\cosh at$	$\frac{s}{s^2-a^2}$
$\sinh at$	$\frac{a}{s^2-a^2}$

LAPLACE TRANSFORMS