JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY

SCHOOL OF MATHEMATICS AND ACTURIAL SCIENCE

UNIVERSITY EXAMINATION FOR DEGREE OF MASTER OF SCIENCE IN PURE MATHEMATICS

1st YEAR 1st SEMESTER 2018/2019 ACADEMIC YEAR

MAIN REGULAR

COURSE CODE: SMA 801

COURSE TITLE: ABSTRACT INTEGRATION I

EXAM VENUE: **STREAM: (Msc. Pure Mathematics)**

DATE: **EXAM SESSION:**

TIME: 3.00HRS

Instructions:

1. **Answer any THREE questions only**
2. **Candidates are advised not to write on the question paper.**
3. **Candidates must hand in their answer booklets to the invigilator while in the examination room**
4.

QUESTION ONE [20 MARKS]

(a) Describe CBS inequality and show its applications in integration theory. (7 marks)
(b). Show that a measure is finitely additive. (6 marks)
(c). Describe the relevance of integral calculus to other fields of mathematics. (7 marks)

QUESTION TWO [20 MARKS]

State and prove the following theorems and hence give their applications in other fields:
(a). Monotone Convergence Theorem. (10 marks)
(b). Cantor's intersection Theorem. (10 marks)

QUESTION THREE [20 MARKS]

(a). Define the following terms giving relevant examples.
 (i). Sigma-finite measure space. (5 marks)
 (ii). Probability measure. (5 marks)
(b). Show that the length of an interval is equal to its outer measure. (10 marks)

QUESTION FOUR [20 MARKS]

(a). Show that any non-degenerate interval of **R** is uncountable. (6 marks)

(b). State and prove Vitali's Covering Theorem. (6 marks)

(b). Describe the terms: Subcover, Outer measure, Lower Riemann integral and Measure space. (8 marks)

QUESTION FIVE [20 MARKS]

(a). Prove that a measure is countably additive. (6 marks)

(b). State and prove Fatou's Lemma. Moreover, describe its consequences. (6 marks)

(c). State and prove the Lebesgue's Dominated Convergence Theorem. (8 marks)