JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY

SCHOOL OF MATHEMATICS AND ACTUARIAL SCIENCE

UNIVERSITY EXAMINATION FOR DEGREE OF MASTER OF SCIENCE

IN PURE MATHEMATICS

1ST YEAR 1ST SEMESTER 2018/2019 ACADEMIC YEAR

MAIN CAMPUS

COURSE CODE: SMA 803

COURSE TITLE: FUNCTIONAL ANALYSIS I

EXAM VENUE: **STREAM: (MSC PURE MATHEMATICS)**

DATE: **EXAM SESSION:**

TIME: 3.00HRS

Instructions:

- **Answer any 3 questions only.**
- **Candidates are advised not to write on the question paper.**
- **Candidates must hand in their answer booklets to the invigilator while in the examination room.**
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1(i) Let (X, d) be a metric space and Y a subset of X. Show that d_Y is a metric on Y and (Y, d_Y) is a subspace of (X, d). **[10 marks]**

(ii) Prove that if E is a normed space and M is a closed linear subspace of E such that E and E/M are Banach spaces then E is a Banach space. **[10 marks]**

2 (i) Prove that every Cauchy sequence is bounded but the converse need not be true. **[6 marks]**

(ii) Prove that every convergent sequence is Cauchy but the converse need not be true. **[7 marks]**

(iii) Using the axiom of choice, show the existence and uniqueness of unbounded linear functional. **[7 marks]**

3 (i) Construct an example of a series in a Hilbert space that converges unconditionally but not absolutely. **[6 marks]**

(ii) State and prove Cantor's intersection theorem. **[7 marks]**

(iii) State and prove Banach's fixed point theorem. **[7 marks]**

4 (i) Show that an arbitrary union of open sets is open. **[7 marks]**

(ii) Show that a set is open if its complement is closed. **[7 marks]**

(iii) Prove that a mapping T taking a Hilbert space H to its dual H* defined by $T(h) = L_h$ is an anti-linear isometric bijection of H onto H* if L_h takes H onto K. **[6 marks]**

5 (i) Describe the terms: Metric space, Neigbourhood, Open ball and Closed set. **[10 marks]**

(ii) State and prove Weiener's theorem. **[10 marks]**