JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY

SCHOOL OF MATHEMATICS AND ACTUARIAL SCIENCE

UNIVERSITY EXAMINATION FOR DEGREE OF MASTER OF SCIENCE
1ST YEAR 1ST SEMESTER 2018/2019 ACADEMIC YEAR

KISUMU CAMPUS

COURSE CODE: SMA 817
COURSE TITLE: ORDINARY DIFFERENTIAL EQUATIONS I

EXAM VENUE: **STREAM: MSC SCIENCE Y1S1**

DATE: **EXAM SESSION:**

TIME: 3.00 HOURS

Instructions:

1. **Answer any THREE questions.**
2. **Candidates are advised not to write on the question paper.**
3. **Candidates must hand in their answer booklets to the invigilator while in the examination room.**

QUESTION ONE (20 MARKS)

a) Solve the differential equations

i) $x(1-x^2)\frac{dy}{dx}+(2x^2-1)y=x^3\sqrt{y}$ (7 marks)

ii) $\frac{d^2y}{dx^2}-4\frac{dy}{dx}+9y-4e^x=0$ (6 marks)

b) Show that for a second order differential equation of the form $a\frac{d^2y}{dx^2}+b\frac{dy}{dx}+cy=f(x)$ for $f(x)\neq 0$ and taking $y=c_1u_1(x)+c_2u_2(x)$ to be a solution of $a\frac{d^2y}{dx^2}+b\frac{dy}{dx}+cy=0$, then by replacing arbtrary constants c_1 and c_2 by $v_1(x)$ and $v_2(x)$ then we could solve the pair of simultaneous equations

$$v_1'u_1+v_2'u_2=0$$
$$v_1'u_1'+v_2'u_2'=f(x)$$

(7 marks)

To obtain the solution to the particular integral $y=u_1v_1+u_2v_2$

QUESTION TWO (20 MARKS)

a) Show that $u_1=x$ is a solution to the differential equation $(1-x^2)\frac{d^2y}{dx^2}-2x\frac{dy}{dx}+2y=0$, hence use the reduction of order method to find the second linearly independent solution $u_2(x)$ (8 marks)

b) Find all the solutions of the initial value problem $\dot{X}=\begin{pmatrix}3 & 1 & -1\\ 1 & 3 & -1\\ 3 & 3 & -1\end{pmatrix}\vec{X}$

$$x(0)=\begin{pmatrix}1\\ -2\\ -1\end{pmatrix}$$

(12 marks)

QUESTION THREE (20 MARKS)

Given the system of first order ordinary differential equation

$$\frac{dx}{dt} = x - y + 4z$$

$$\frac{dy}{dt} = 3x + 2y - z$$

$$\frac{dz}{dt} = 2x + y + z$$

a) Express the system in the matrix form $\underline{\dot{X}} = A\underline{X}$ (2 marks)

b) Show that $v_1 = \begin{pmatrix} -1 \\ 4 \\ 1 \end{pmatrix}, v_2 = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$ are eigenvectors of A hence find the third eigenvector (7 marks)

c) Determine $\Phi(t)$, the fundamental matrix of the system (6 marks)

d) Obtain $\underline{X}$ the general solution of the system (5 marks)

QUESTION FOUR (20 MARKS)

Given the system of nonlinear differential equations

$$\frac{dx}{dt} = -2xy$$

$$\frac{dy}{dt} = -x + y + xy - y^2$$

(a)Find all its critical points (8 marks)

(b) Determine the stability nature of each of the critical points in part (a) (12 marks)

QUESTION FIVE (20 MARKS)

a) Prove that if $x_1(t)$ and $x_2(t)$ are linearly independent on $L(x) = 0$ on an interval I then the wronskian $W[x_1(t), x_2(t)] \neq 0$ (6 marks)

b) Find e^{At} if $A = \begin{pmatrix} 1 & -1 & -1 \\ 1 & 3 & 1 \\ -3 & 1 & -1 \end{pmatrix}$ (14 marks)