JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY

SCHOOL OF MATHEMATICS AND ACTUARIAL SCIENCE UNIVERSITY

EXAMINATION FOR THE DEGREE OF MASTER OF SCIENCE IN APPLIED MATHEMATICS

1ST YEAR 1ST SEMESTER 2018/2019 ACADEMIC YEAR

KISUMU CAMPUS

COURSE CODE: SMA 839 | **COURSE TITLE: NUMERICAL ANALYSIS I**

EXAM VENUE: | **STREAM: (BSc. Actuarial)**

DATE: | **EXAM SESSION:**

TIME: 3.00 HOURS

Instructions:

1. **Answer ANY 3 questions**
2. **Candidates are advised not to write on the question paper.**
3. **Candidates must hand in their answer booklets to the invigilator while in the examination room.**

QUESTION ONE (20 MARKS)

a) Given

$$P_n(x) = f_i^{(0)} + (x - x_0) f_i^{(1)} + (x - x_0)(x - x_1) f_i^{(2)} + \ldots + (x - x_0)(x - x_1)\ldots(x - x_{n-1}) f_i^{(n)}$$

verify that $P_n(x)$ passes exactly through the data point x_2. (6 marks)

b) Consider the set of discrete points below which satisfy the function $f(x) = \frac{2}{x}$

x	$f(x)$
1.20	1.666667
1.30	1.538461
1.40	1.428571
1.45	1.379310
1.50	1.333333
1.60	1.250000
1.70	1.176471

(i) Give the nth degree Lagrange polynomial $P_n(x)$ which passes through $n+1$ points. Hence use it to find $P_3(1.38)$. (7 marks)

(ii) Form six-place difference table for the above data points and use it to calculate $P_3(1.68)$ by the Newton backward-difference polynomial. (7 marks)

QUESTION TWO (20 MARKS)

a) Give Gauss elimination procedure in a format suitable for computer programming. (8 marks)

b) Solve the following system of linear algebraic equations by applying the Gauss-Jordan elimination method

$$\begin{aligned}
2x_1 - 2x_2 + 2x_3 + x_4 &= 7 \\
2x_1 - 4x_2 + x_3 + 3x_4 &= 10 \\
-x_1 + 3x_2 - 4x_3 + 2x_4 &= -14 \\
2x_1 + 4x_2 + 3x_3 - 2x_4 &= 1
\end{aligned}$$

(12 marks)

QUESTION THREE (20 MARKS)

Find the solution of the following matrix equation using Successive Over-Relaxation method for $\omega = 1.05$ with initial guess of $x^{(0)T} = \begin{bmatrix} 0.0 & 0.0 & 0.0 & 0.0 \end{bmatrix}$:

$$\begin{bmatrix} 4 & -1 & 0 & 0 \\ -1 & 4 & -1 & 0 \\ 0 & -1 & 4 & -1 \\ 0 & 0 & -1 & 4 \end{bmatrix} \begin{bmatrix} x_i \end{bmatrix} = \begin{bmatrix} 150 \\ 200 \\ 150 \\ 100 \end{bmatrix}$$

(*Conduct five iterations*)

QUESTION FOUR (20 MARKS)

Solve for the largest (in magnitude) eigenvalue of matrix, $\begin{bmatrix} 1 & 1 & 2 \\ 2 & 1 & 1 \\ 1 & 1 & 3 \end{bmatrix}$ and the corresponding eigenvector x by the power method.

a) Let the first component of x be the unity component. (5 marks)
b) Let the second component of x be the unity component. (5 marks)
c) Let the third component of x be the unity component. (5 marks)
d) Show that the eigenvectors obtained in parts (a), (b), and (c) are equivalent. (5 marks)

QUESTION FIVE (20 MARKS)

a) Use the contiuous Fourier series to approximate the square or rectangular wave function:

$$f(t) = \begin{cases} -1 & -T/2 < t < T/2 \\ 1 & T/2 < t < T/4 \\ -1 & T/4 < t < T/2 \end{cases}$$

(10 marks)

b) Given $y = 1.7 + \cos(4.189t + 1.0472)$. Generate 10 discrete values for the function at intervals of $\Delta t = 0.15$ for the range $t = 0$ to $t = 1.35$. Use this information to evatuate the coefficient of $y = A_0 + A_1 \cos(\omega_0 t) + B_1 \sin(\omega_0 t) + e.$ (10 marks)