JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE & TECHNOLOGY UNIVERSITY EXAMINATIONS 2012/2013

1ST YEAR 1ST SEMESTER EXAMINATION FOR THE DEGREE OF MASTERS OF PURE AND APPLIED MATHEMATICS

(SCHOOL BASED-MAIN)

COURSE CODE: SMA 818

COURSE TITLE: ORDINARY DIFFERENTIAL EQUATIONS II

DATE: 26 /8/13 TIME: 9.00 - 12.00 NOON

DURATION: 3 HOURS

INSTRUCTIONS

1. **This paper contains five (5) questions.**
2. **Answer ANY other THREE questions.**
3. **Write all answer in the booklet provided.**
4. **Show ALL your workings.**

QUESTION ONE (20 MARKS)

Consider a Bessel Equation of order n given by $x^2 \dfrac{d^2 y}{dx^2} + x\dfrac{dy}{dx} + \left(x^2 - n^2\right)y = 0$

a) By assuming a solution $y = a_0 x^m + a_1 x^{m+1} + a_2 x^{m+2} + ...$ show that the roots of the indical equation are $m = n$ and $m = -n$. (3 marks)

b) From *a*) above use $m = n$ and $m = -n$ to obtain the possible bessel functions (3 marks)

c) Considering non integral and non zero values of *n* determine the complete solution of the Bessel's equation giving your answer in terms of Γ (gamma) (5 marks)

d) Taking $J_n(x) = \sum_{r=0}^{\infty} (-1)^r \left(\dfrac{x}{2}\right)^{n+2r} \dfrac{1}{r!\Gamma(n+r+1)}$ and leting the solution be $y = u(x)J_n(x)$ for integral values of *n* Show that the complete solution is $y = AJ_n(x) + BJ_n(x)\int \dfrac{dx}{x\left[J_n(x)\right]^2}$ (9 marks)

QUESTION TWO (20 MARKS)

Solve in series $(1 - x^2)\dfrac{d^2 y}{dx^2} - x\dfrac{dy}{dx} + 2y = 0$

QUESTION THREE (20 MARKS)

Use frobenius method to solve

$$x\frac{d^2 y}{dx^2} + \frac{dy}{dx} + xy = 0$$

QUESTION FOUR (20 MARKS)

a) Define Orthogonality (3 marks)

b)Given the Sturm Liouville problem $y'' + \text{ª}y = 0$, $y(0) = 0$ and $y(\text{®}) = 0$
Find the eigen function and verify its orthogonality (17 marks)

QUESTION FIVE (20 MARKS)

a) Determine the constants $a_1, a_2\ a_3$, so that $f(x) = a_1 x + 2, a_1,$ $g(x) = a_2 x^2 + a_3 x + 1$ and $h(x) = x - 1$ are mutually orthogonal in $0 \le x \le 1$ and then obtain the corresponding orthonormal set (12 marks)

b) Solve the boundary value problem $y'' + 4y' + (4 + 9a)y = 0$, $y(0) = 0$, $y(l) = 0$ (8 marks)