JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY

SCHOOL OF MATHEMATICS AND ACTUARIAL SCIENCE

UNIVERSITY EXAMINATION FOR DEGREE OF BACHELOR OF SCIENCE IN ACTUARIAL SCIENCE

3^{RD} YEAR 2^{ND} SEMESTER 2018/2019 ACADEMIC YEAR

MAIN REGULAR

COURSE CODE: SAC 304

COURSE TITLE: ACTUARIAL LIFE CONTINGENCIES I

EXAM VENUE: **STREAM: (BSc. Actuarial)**

DATE: **EXAM SESSION:**

TIME: 2.00 HOURS

Instructions:

1. **Answer question 1 (Compulsory) and ANY other 2 questions**
2. **Candidates are advised not to write on the question paper.**
3. **Candidates must hand in their answer booklets to the invigilator while in the examination room.**

QUESTION 1 [COMPULSORY] [30 Marks]

a) Give an expression using random a variable, for the expected present value of an annual temporary annuity-due. [**3 Marks**]

b) Calculate on the basis of AM92 mortality table and a rate of interest of 4% p.a

(i) ${}_{10}p_{73}$ [**3 Marks**]

(ii)$A_{64:\overline{10}|}$ [**3 Marks**]

(c)A life aged 60 has purchased an annuity that provides payments at the start of each year for the next 5 years. The amount of first payment is Kshs. 10,000. Each subsequent payment is Kshs.2,000 higher than the last. Assuming that mortality follows that in tables and $i = 0.05$, calculate the expected present value of the annuity. [**5 Marks**]

d) You are given the following extract from a select and ultimate mortality table with a 2-year select period.

x	$l_{[x]}$	$l_{[x]+1}$	l_{x+2}	$x+2$
50	80,625	79,954	78,839	52
51	79,137	78,402	77,252	53
52	77,575	76,770	75,578	54

Given that $i = 0.05$, Calculate

(i) $A_{[51]:\overline{3}|}$ [**3 Marks**]

(ii) $\ddot{a}_{[52]:\overline{3}|}$ [**3 Marks**]

e) A whole life insurance is issued on a life aged 20 with the benefit to be paid at death. If death occurs prior to age 40 the benefit is 1, whereas it is 2 if death occurs at age 40. The effective annual rate of interest is 5% mortality is assumed to follow the model.

$$l_x = 100 - x \quad \text{for all ages } x \leq 100.$$

Let Z be the random present value of the benefit at issue.

Calculate

(i) E(Z) [**5 Marks**]

(ii) Var(Z) [**5 Marks**]

QUESTION 2

a) Define the following life assurance contracts:

(i) Whole life assurance. **[2 Marks]**

(ii) Term assurance. **[2 Marks]**

(iii) Endowment assurance. **[2 Marks]**

(iv) Pure Endowment assurance. **[2 Marks]**

b) The present value Z of an endowment assurance of the benefit amount of 1 is

$$Z = \begin{cases} V^{k+1} & k = 0, 1, ..., n-1 \\ V^{n} & k = n, n+1, ... \end{cases}$$

Derive the expression for the net single premium for this payment.

[4 Marks]

c) A life aged 50 who is subject to mortality of the A 1967-70 select table effects a pure endowment policy with term of 20 years for the sum assured of Kshs. 100,000.

(i) Write down the present value of the benefits under this contract, regarded as a random-variable. **[2 Marks]**

(ii)Assuming an effective interest rate of 5% annum, calculate the mean and the variance of the present value of the benefits under this contract.

[6 Marks]

QUESTION 3

Let Z_1, be the present value random variable for an n-year term insurance of 1 on (x), and let Z_2 be the present value random variable for a n-year endowment insurance of 1 on (x). Claims are payable at the moment of death.

Given

$$\begin{aligned} V^n &= 0.250 \\ {}_nP_x &= 0.400 \\ E(Z_2) &= 0.400 \\ Var(Z_2) &= 0.055 \end{aligned}$$

Calculate

(a) E(Z_1) **[10 Marks]**

(b) Var(Z_1) **[10 Marks]**

QUESTION 4

A fully continuous level premium 10 year term insurance issued to (x) pays a benefit of 1 plus the return of all premiums paid accumulated with interest. The interest rate used in calculating the death benefit is the same as that used to determine the present value of the insurer's loss. Let G denote the rate of annual premium paid continuously.

a) Write down an expression for the insurer's loss random variable.

[**6 Marks**]

b) Derive an expression for Var(L). [**8 Marks**]

c) Show that, if G is determined by the equivalence principle, then

$$Var(L) =^2 \bar{A}^1_{x:\overline{10}|} + \frac{(\bar{A}^1_{x:\overline{10}|})^2}{{}_{10}P_x}$$

The pre-superscript indicates that the symbol is evaluated at the force of interest of 2δ where δ is the force of interest underlying the usual symbols.

[**6 Marks**]

QUESTION 5

a) Define ${}_tP(A_{x:\overline{n}|})$ and determine it's expressions in terms of assurance and annuity functions. [**6 Marks**]

b) A fully discrete 20 year endowment insurance of 1 is issued to a life aged 40. The assurance also provides for the refund to all net premiums paid accumulated at the interest rate if death occurs within 10 years of issue. Present values are calculated at the same interest rate i. Using the equivalence principle, the net annual premium for 20 years of this policy can be written in the form

$$\frac{A_{40:\overline{20}|}}{k}$$

Determine k. [**14 Marks**]