JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY

DRAFT - EXAMINATIONS 2012/2013

SEMESTER 1 FIRST YEAR MSc EXAMS

COURSE CODE: SMA 839
COURSE TITLE: NUMERICAL ANALYSIS I

DATE : Aug, 2013 TIME: 3hrs

INSTRUCTIONS

ATTEMPT ANY **THREE** QUESTIONS

Show all the necessary working

Question1 [20 marks]

Given the real matrices $M = \begin{bmatrix} 3 & 1 & -2 & -1 \\ 2 & -2 & 2 & 3 \\ 1 & 5 & -4 & -1 \\ 3 & 1 & 2 & 3 \end{bmatrix}$ $\underline{b} = \begin{bmatrix} 30 \\ -80 \\ 30 \\ 0 \end{bmatrix}$

(i) Determine if M is real symmetric matrix.

(ii) Use Doolittle's method to factorize M into lower and upper triangular form $M = LU$.

(iii) Use the factorized form of M to solve the system of linear equations $M\underline{X} = \underline{b}$. **[20 marks]**

Question2 [20 marks]

Consider the system of nonlinear equations

$$f(x,y) = x^2 + y^2 - 1 = 0$$

$$g(x,y) = x^2 - y^2 + \frac{1}{2} = 0$$

(a)Derive the improved Newton's iterative scheme

$$x_{n+1} = x_n - \frac{g(x_n, y_n) + f(x_n, y_n)}{4x_n}$$

$$y_{n+1} = y_n + \frac{f(x_{n+1}, y_n) - g(x_{n+1}, y_n)}{-4y_n}$$

for the system. **[8 marks]**

(b)Apply six times the improved Newton's iterative scheme to obtain the approximate solution of the system. On the same table display the results ; $n, x_n, y_n, f(x_n, y_n), g(x_n, y_n)$.

Take the initial root as $(x_0, y_0) = (1,3)$ **[12 marks]**

Question.3 [20 marks]

For the nonlinear equation; $x^3 - x - 6 = 0$, develop the five possible fixed-point iterative formulas. Determine explicitly which of the formulas are likely to converge to a solution of the above nonlinear equation, taking the initial solution as $x_0 = 2.5$

Question.4 [20 marks]

(a) Use the data below to construct a complete divided difference table. Determine an interpolating polynomial $p(x)$ for the function $f(x)$ and hence approximate $f(1.3)$.

x :	1	1.5	1.75	2	1.1
$f(x)$:	0.000	.40547	.55962	.69315	0.09531

[15 marks]

(b) If $f(x) = In\,x$, calculate error bound for $f(1.3)$ and show that the approximation to $f(1.3)$

satisfies this error bound. **[5 marks]**

Question.5 [20 marks]

Let the $n \times n$ matrix $A = \begin{pmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & \ddots & \vdots \\ a_{n1} & \cdots & a_{nn} \end{pmatrix}$ have eigenvalues a_i and linearly independent eigenvectors x_i .

(a) Derive an algorithm for approximation of the dominant eigenvalue a_1 of A .
Describe precisely the computation procedure. **[8 marks]**

(b) Consider a three by three matrix A, of linear transformation from R^3 into itself given by

$$A = \begin{bmatrix} 3 & 0 & 1 \\ 2 & 2 & 2 \\ 4 & 2 & 5 \end{bmatrix}$$

(i) Apply six times the power method to approximate the dominant eigenvalue a_1 of matrix A and v_1 the corresponding eigenvector., working with at least six decimal places.

(ii) Given that $a_m = 2$ is also an eigenvalue of A, show that a_1, a_*, a_m do lie in the interval $\left[-\|A\|_E , \|A\|_E \right]$. [1 2marks]