JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE & TECHNOLOGY UNIVERSITY EXAMINATIONS 2012/2013

1ST YEAR 2ND SEMESTER EXAMINATION FOR THE MASTER IN SCIENCE (PURE AND APPLIED MATHEMATICS)

(KISUMU LEARNING CENTRE)

COURSE CODE: SMA 862

COURSE TITLE: PARTIAL DIFFERENTIAL EQUATIONS III

DATE: 29/8/13 **TIME: 9.00 – 12.00 NOON**

DURATION: 3 HOURS

INSTRUCTIONS

1. **This paper contains SIX (6) questions.**
2. **Answer any FOUR questions.**
3. **Start each question on a fresh page.**
4. **All workings must be shown clearly. Write all answer in the booklet provided.**

QUESTION ONE (15 marks)

a) Find the characteristics of the equation:

$$3u_{xx} + 10u_{xy} + 3u_{yy} = 0$$

and reduce it to the appropriate standard form and obtain the general solution. [7 marks]

b) Determine the type of the following equation:

$$u_{xx} + u_{yy} = 0$$

and after reducing it to the hyperbolic form, deduce the formula:

$$u(x, y) = \frac{1}{2}¤(x + iy) + \frac{1}{2}\bar{¤}(x, iy).$$

express any harmonic function u as the real part of some analytic function ¤ of complex variable $(x + iy)$. [8 marks]

QUESTION TWO (15 marks)

Give the D'Alembert's solution of the one-dimensional wave equation

$$u_{tt} = c^2 u_{xx}$$

where u is the dependent variable, x and t are the independent variables and c^2 is a parameter the dimension of c being the speed. [15 Marks]

QUESTION THREE (15 marks)

Consider the heat conduction in a thin metal bar of length L with insulated sides. Let us suppose that the end $x = 0$ is held at u_0 and the end $x = L$ is held at u_L degrees Celsius for all time $t > 0$. Let us suppose that the temperature distribution at $t = 0$ is $u(x, 0) = f(x),\ 0 \le x \le L.$ Determine the temperature distribution in the bar at any position at any time $t > 0$. [15 marks]

QUESTION FOUR (15 marks)

Given a Laplace's equation: $u_{xx} + u_{yy} = 0$, where $u(x, y)$ represents the velocity of a fluid particle in a certain domain, determine $u(x, y)$ inside a unit circle, $x^2 + y^2 < 1,$ when its values on the circumference $x^2 + y^2 = 1,$ are prescribed. [15 Marks]

QUESTION FIVE (15 marks)

a) Determine for what values of x and y the equation:

$$(1 + y)u_{xx} + 2(1 - x)u_{xy} + (1 - y)u_{yy} = u$$

is (i) hyperbolic (ii) parabolic or (iii) elliptic [3 marks]

b) Solve the equation:

$$\frac{\partial u}{\partial x} = 2\frac{\partial u}{\partial t} + u, \text{ given } u(x, 0) = 6e^{-3x}.$$ [4 marks]

c) Obtain the solution of the equation:

$$\frac{\partial^2 u}{\partial x^2} - 2\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 0$$

by the method of separation of variables. [8 Marks]

QUESTION SIX (15 marks)

A string is stretched and fastened to two points l apart. Motion is started by displacing the string in the form

$$y = a\sin\frac{\text{®}x}{l}.$$

From which it is released at time $t = 0$. Show that the displaced of any point at a distance x from one end at time t is given by

$$y(x,t) = a\sin\frac{\text{®}x}{l}\cos\frac{\text{®}ct}{l}.$$

[15 Marks]