JARAMOGI OGINGA ODINGA UNIVERSITY
OF SCIENCE AND TECHNOLOGY

FIRST YEAR SECOND SEMESTER EXAMINATION FOR THE DEGREE OF MASTERS OF ……………

SMA *862*: PARTIAL DIFFERENTIAL EQUATIONS III

Date: …….. December, 2013 ***Time: ……. - ………***

INSTRUCTIONS:

1. This examination paper contains five questions. Answer **any three questions**.
2. Start each question on a fresh page.
3. All the working must be shown clearly.
4. Indicate question number clearly at the top of each page.

QUESTION ONE (20 marks)

a) Determine for what values of x and y the equation

$$(x+2)u_{xx}+2xu_{xy}+yu_{yy}=2x+y$$

is (i) hyperbolic (ii) parabolic or (iii) elliptic [6 marks]

b) Use the method of separation of variables to solve

$$3\frac{\partial u}{\partial x}+2\frac{\partial u}{\partial y}=0, \text{ where } u(x,0)=4e^{-x}$$ [7 marks]

c) Find a solution of the equation

$$\frac{\partial^2 u}{\partial x^2}=\frac{\partial u}{\partial y}+2u$$

in the form $u=f\left(x\right)g\left(y\right)$ subject to the conditions $u=0$

and $\frac{\partial y}{\partial x}=1+e^{-3y}$, when $x=0$ for all values of y. [7 marks]

QUESTION TWO (20 marks)

Consider the heat conduction in a thin metal bar of length L with insulated sides. Suppose that the ends $x=0$ and $x=L$ are held at temperature $u=0^o c$ for all time $t>0$. In addition let us suppose that the temperature distribution at $t=0$ is $u(x,0)=f(x),\ 0\le x\le L$. Determine the temperature distribution in the bar at some subsequent time $t>0$ [20 marks]

QUESTION THREE (20 marks)

Consider the one-dimensional wave equation

$$u_{tt}=c^2u_{xx}$$

where u is the dependent variable, x and t are the independent variables and c^2 is a parameter the dimension of c being the speed. Give the D'Alembert solution of this equation explaining the physical interpretation of the solution. [20 Marks]

QUESTION FOUR (20 marks)

Solve the two-dimensional wave equation

$$\frac{\partial^2 u}{\partial t^2}=c^2\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)$$

given the boundary conditions

$$u\left(0,y,t\right)=u\left(1,y,t\right)=u\left(x,0,t\right)=u\left(x,1,t\right)=0$$

and the initial conditions

$$u\left(x,y,0\right)=f\left(x,y\right)=k\sin 2® x\sin ® y$$

and $\frac{\partial u}{\partial t}=0$ when $t=0$ [20 Marks]

QUESTION FIVE (20 marks)

a) Determine the type of the following equation

$$u_{xy} + u_{yy} = 0$$

and after reducing it to the hyperbolic form, deduce the formula

$$u(x, y) = \frac{1}{2} ¤ (x + iy) + \frac{1}{2} \overline{¤} (x + iy)$$

expressing any harmonic function u as the real part of some analytic function ¤ of complex variable $(x + iy)$ [10 Marks]

b) Find the current i and voltage e in a line of length L, t seconds after the ends are suddenly grounded, given that

$$i(x,0) = i_0,\ e(x,0) = e_0 \sin \frac{® x}{L}$$

also R and G are negligible [10 Marks]