JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY
SCHOOL OF MATHEMATICS AND ACTUARIAL SCIENCE

UNIVERSITYDRAFT EXAMINATION FOR BSc/BEd IN MATHEMATICS

4th YEAR 1st SEMESTER 2018/2019 ACADEMIC YEAR

(INSTITUTIONAL BASED)

MAIN CAMPUS

COURSE CODE: SMA405

COURSE TITLE: PARTIAL DIFFERENTIAL EQUATIONS I

EXAM VENUE: **STREAM: BSc Y4S1**

DATE: 24/08/19

TIME: 2 HOURS **EXAM SESSION: 3.00 – 5.00PM**

Instructions:

Answer question1 and any other two questions

1. **Show all the necessary working**
2. **Candidates are advised not to write on the question paper**
3. **Candidates must hand in their answer booklets to the invigilator while in the examination room**

QUESTION ONE(30 MARKS) -COMPULSORY

(a) Explain the three main properties of classifying the partial differential equation

$$a\frac{\partial^2 u}{\partial x^2}+b\frac{\partial^2 u}{\partial x\partial y}+c\frac{\partial^2 u}{\partial y^2}+d\frac{\partial u}{\partial x}+e\frac{\partial u}{\partial y}+fu+g=0$$ (6Mks)

(b) If the points $p(a,b)$ of the curve $g(x,y)=4x^2y-y^2-8x^2-2x^4+1400$ satisfy system of the equations $\frac{\partial g}{\partial x}=0$, $\frac{\partial g}{\partial y}=0$

discuss the nature of such point $p(a,b)$ (5Mks)

(c) Prove that the Pfaffian differential equation

$(y^2+yz)dx+(xz+z^2)dy+(y^2-xy)dz=0$ is integrable (4Mks)

(d) Categorize the given partial differential equations below;

(i) $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+xy=0$ (ii) $\frac{\partial u}{\partial t}=1002\frac{\partial^2 u}{\partial x^2}$ (iii) $\frac{\partial^2 u}{\partial x^2}+y^3\frac{\partial^2 u}{\partial y^2}=0$

(iv) $\frac{\partial u}{\partial t}+t\frac{\partial^2 u}{\partial t\partial x}=\frac{\partial^2 u}{\partial x^2}$ [6 marks]

(e) State degree , order of the partial differential equations below;

(i) $\frac{\partial^2 u}{\partial x^2}+\sqrt{\frac{\partial^2 u}{\partial y^2}}+2y^4=0$ (ii) $\left(\frac{\partial u}{\partial t}\right)^9=1002\frac{\partial u}{\partial x}$ (iii) $\frac{\partial^2 u}{\partial x^2}+y^3\frac{\partial^2 u}{\partial y^2}=0$

and identify which of the equations are linear [9 marks]

QUESTION TWO (20Mks)

Show that the differential equation $\left(x^2z-y^3\right)dx+3xy^2dy+x^3dz=0$ is

i) homogenous **(5 Mks)**
ii) integrable **(10 Mks)**
iii) in Pfaffian form

and solve it. **(5Mks)**

QUESTION THREE (20 Mks)

Given the curve $F(x,y,z)=8x^2+24y^2+16z^2+24x+16z-10$

i) determine and classify all the critical points **(10 Mks)**

ii) obtain the minimum and maximum values of F **(10 Mks)**

QUESTION FOUR (20 Mks)

Consider a perfectly flexible elastic string, stretched between two points at $x=0$ and $x=1$ with uniform tension τ.

If the string is displaced slightly from its initial position while the ends remain fixed, and then released, the string will oscillate. The position u in the string at any instant will then be a function of its distance from one end x, of the string and also of time t $u=u(x,t)$,

i) Show that the equation of the motion is given by the partial differential equation

$$\frac{\partial^2 u}{\partial x^2}=\frac{\partial^2 u}{\partial t^2}$$

ii) Solve the equation $\frac{\partial^2 u}{\partial x^2}=\frac{\partial^2 u}{\partial t^2}$ given

the boundary conditions

$u(0,t)=u(1,t)=0$ for all time $t\ge 0$

and the initial condition

$u(x,0)=\sin 2\pi$, $u_t(x,0)=0$ (20 marks)

QUESTION FIVE (20Mks)

Given the initial boundary value pde heat equation

$u_t=u_{xx}$, $0<x<1, t>0$

satisfying the conditions

$u(0,t)=1,\ u(1,t)=1\quad 0<x<1, t>0$

$$u(x,0)=1+\cos 2\pi x,\quad 0<x<1$$

Apply variable separation of the form $u(x,t)=X(x)T(t)$.

to solve the pde.

(20 mark)