JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY
SCHOOL OF MATHEMATICS ,ACTUARIAL SCIENCE AND BPS

UNIVERSITYDRAFT EXAMINATION FOR BSc/BEd IN MATHEMATICS

SUPPLEMENTARY/SPECIAL

4^{th} YEAR 1^{st} SEMESTER 2019/2020ACADEMIC YEAR

MAIN CAMPUS

COURSE CODE: SMA405
COURSE TITLE: Partial Differential Equations I

EXAM VENUE: AUDITORIUM **STREAM: BSc Y4S1**

TIME: 2 HOURS **EXAM SESSION:**

Instructions:

Answer question1 and any other two questions

1. **Show all the necessary working**
2. **Candidates are advised not to write on the question paper**
3. **Candidates must hand in their answer booklets to the invigilator while in the examination room**

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Question 1:(30MKS) COMPULSORY

(a) Given the function

$F(x,y)=4x^2y-y^2-8x^2-2x^4+10$

(i).Find $\frac{\partial F}{\partial x}$, $\frac{\partial F}{\partial y}$,

(ii) Determine all the stationary points of F

(iii) Find $\frac{\partial^2 F}{\partial x^2}$, $\frac{\partial^2 F}{\partial y^2}$ $\frac{\partial^2 F}{\partial x \partial y}$

(iv) Determine the nature of the stationary points of F (18mks)

(b) Given the partial differential equation

(i) $x\frac{\partial F}{\partial x}-y\frac{\partial F}{\partial y}=3xy^2$

(ii) $x^2\frac{\partial^2 F}{\partial x^2}-y^2\frac{\partial^2 F}{\partial y^2}+x\frac{\partial F}{\partial x}-y\frac{\partial F}{\partial y}=0$

(iii) $x^2\frac{\partial^3 F}{\partial x^3}-y^2\left(\frac{\partial^2 F}{\partial y^2}\right)^4+x\frac{\partial F}{\partial x}-y\frac{\partial F}{\partial y}=0$

State in each the ; ORDER, DEGREE and whether **LINEAR or NONLINEAR. (12mks)**

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Question 2:(20MKS) Solve the homogeneous the partial differential equation

(i) $\left(D^2-DD'-6D'^2\right)u=0$

(ii) $\left(4D^2-12DD'+9D'^2\right)u=0$

Question 3. :(20MKS)

Solve the inhomogeneous the partial differential equation

(i) $\left(D^2-3DD'-4D'^2\right)u=e^{x+2y}$

(ii) $\left(D^2-DD'-6D'^2\right)u=\sin x\cos 2y$

Question 4: **:(20MKS)**

Solve the partial differential equation $x^2\frac{\partial^2 F}{\partial x^2}-y^2\frac{\partial^2 F}{\partial y^2}+x\frac{\partial F}{\partial x}-y\frac{\partial F}{\partial y}=0$

Use the change of variables from x , y to u , v where $u=xy$, $v=\frac{y}{x}$

Question 5: :(20MKS)

Consider a perfectly flexible elastic string ,stretched between two points at $x=0$ and $x=1$ with uniform tension τ .

If the string is displaced slightly from its initial position while the ends remain fixed , and then released , the string will oscillate . The position P in the string at any instant will then be a function of its distance from one end (x,) of the string and also of time (t) i.e. $u=u(x,t)$,

. The equation of the motion is given by the partial differential equation

$$\frac{\partial^2 u}{\partial x^2}=\frac{\partial^2 u}{\partial t^2} .$$

Using variable separation, of the form $u(x,t)=X(x)T(t)$

(a) Show that the variables X , T satisfy the ordinary differential equations

$$\frac{1}{X}\frac{d^2X}{dx^2}=\frac{1}{T}\frac{d^2T}{dt^2}$$

(b) determine the displacement $u(x,t)$ given

the boundary conditions

$u(0,t)=u(1,t)=0$ for all time $t \geq 0$

and the initial condition

$u(x,0)=0$