JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY

SCHOOL OF MATHEMATICS AND ACTUARIAL SCIENCE

UNIVERSITY EXAMINATION FOR DEGREE OF BACHELOR OF SCIENCE ACTUARIAL

SPECIAL RESIT 2 2020/2021 ACADEMIC YEAR

MAIN REGULAR

COURSE CODE: SMA 102

COURSE TITLE: Calculus I

EXAM VENUE: **STREAM: (BSc. Actuarial)**

DATE: **EXAM SESSION:**

TIME: 2.00 HOURS

Instructions:

1. **Answer question 1 (Compulsory) and ANY other 2 questions**
2. **Candidates are advised not to write on the question paper.**
3. **Candidates must hand in their answer booklets to the invigilator while in the examination room.**

QUESTION ONE (COMPULSORY) (30 marks)

a) If $f(x) = -x^2 + 3x + 6$, find $\dfrac{f(2+a) - f(2)}{a}$. (4 marks)

b) Explain the meaning of: $\lim\limits_{x\to a} f(x) = L$. (2 marks)

c) Find the limit (if it exists)

$$\lim_{x\to 1} \frac{x^2 - 1}{\sqrt{x} - 1}$$ (4 marks)

d) Determine the point of discontinuity (if any) of the function $f(x)$

$$f(x) = \frac{2x^2 - 3x - 2}{x - 2}$$

If the discontinuity is removable, define the function to make it continuous. (4 marks)

e) Find $\dfrac{d}{dx}\left(3x^2 + 5\right)$ from first principles. (4 marks)

f) If f and g are both differentiable, show that $\dfrac{d}{dx}\left[f(x) - g(x)\right] = \dfrac{d}{dx} f(x) - \dfrac{d}{dx} g(x)$.

(4 marks)

g) Find $\dfrac{dy}{dx}$ when $x = 1$, given $y = 3e^{4x} - \dfrac{5}{2e^{3x}} + 8\ln 5x$. Give the answer correct to 3 significant figures. (4 marks)

h) $v = 50\sin 40t$ volts represent an alternating voltage where t is the time in seconds. At a time 20×10^{-3} seconds, find the rate of change of voltage. (4 marks)

QUESTION TWO (20 marks)

a) If $y = \dfrac{2}{\theta^2} + 2\ln 2\theta - 2\left(\cos 5\theta + 3\sin 2\theta\right) - \dfrac{2}{e^{3\theta}}$

(i) $\dfrac{dy}{d\theta}$

(ii) Evaluate $\dfrac{dy}{d\theta}$ when $\theta = \dfrac{\pi}{2}$, correct to 4 significant figures. (6 marks)

b) Prove that if u and v are differentiable, then so is their product uv, and $\dfrac{d}{dx}(uv) = u\dfrac{dv}{dx} + v\dfrac{du}{dx}$.

(5 marks)

c) Evaluate $\lim\limits_{x\to\infty} \dfrac{4x^4 + 5}{\left(x^2 - 2\right)\left(2x^2 - 1\right)}$. Give geometrical interpretation of your solution. (5 marks)

d) For what values of a and b is $f(x) = \begin{cases} ax + 2b, & x \le 0 \\ x^2 + 3a - b, & 0 < x \le 2 \\ 3\text{x} - 5, & x > 2 \end{cases}$

Continuous at every x? (5 marks)

QUESTION THREE (20 marks)

a) Find $D_x f(x)$ given $f(x) = \left(x^3 + 2x\right)e^x$. (5 marks)

b) Using logarithmic differentiation, determine $\frac{dy}{dx}$ given $y = \sqrt[4]{\frac{x^2+1}{x^2-1}}$.

(6 marks)

c) Find y' if $x^y = y^x$. (5 marks)

d) Find the derivative of y with respect to θ given $y = \frac{1+\sin\theta}{\theta+\cos\theta}$. (4 marks)

QUESTION FOUR (20 marks)

a) Use implicit differentiation to find an equation of the tangent to the curve at the point give:
$x^2 + xy + y^2 = 3, \quad (1,1)$. (6 marks)

b) Find $\frac{dy}{dx}$, given $y = \sin(\tan 2x)$. (5 marks)

c) If $x = 2t/(t+2)$, $y = 3t/(t+3)$, find $\frac{dy}{dx}$ in terms of t. (4 marks)

d) Show that the differential equation $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = 0$ is satisfied when $y = xe^{2x}$. (5 marks)

QUESTION FIVE (20 marks) `

a) The parametric equations for a hyperbola are $x = 2\sec\theta$, $y = 4\tan\theta$. Evaluate $\frac{d^2y}{dx^2}$, correct to 4 significant figures, when $\theta = 1$ radian. (6 marks)

b) The displacement s cm of the end of a stiff string at time t seconds is given by: $s = ae^{-kt}\sin 2\pi ft$. Determine the velocity and acceleration of the end of the spring after 2 seconds if $a = 3$, $k = 0.75$ and $f = 20$. (5 marks)

c) Determine the equation of the normal for the curve $y = 2x^2 - 3x$ at the point $(1,2)$. (4 marks)

d) The heat capacity c of a gas varies with absolute temperature as shown:
$c = 26.50 + 7.20\times10^{-3}\theta - 1.20\times10^{-6}\theta^2$.
Determine the maximum value of c and the temperature at which it occurs. (5 marks)