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Volatility Estimation Using European-Logistic Brownian Motion with Jump Diffusion Process

Andanje Mulambula[1], D. B. Oduor[2,*] and B. O. Kwach[1]

1 Department of Mathematics, Kibabii University, Bungoma, Kenya.

2 School of Mathematics and Actuarial Science, Jaramogi Oginga Odinga University of Science and Technology, Bondo, Kenya.

Abstract: Volatility is the measure of how we are uncertain about the future of stock or asset prices. Black-Scholes model formed the foundation of stock or asset pricing. However, some of its assumptions like constant volatility and interest among others are practically impossible to implement hence other option pricing models have been explored to help come up with a much reliable way of predicting the price trends of options. The measure of volatility and good forecasts of future volatility are crucial for implementation, evaluation of asset and derivative pricing of asset. In particular, volatility has been used in financial markets in assessment of risk associated with short-term fluctuations in financial time-series. Constant volatility is not true in practical sense especially in short term intervals because stock prices are able to reproduce the leptokurtic feature and to some extent the "volatility smile". To address the above problem the Jump-Diffusion Model and the Kou Double-Exponential Jump-Diffusion Model were presented. But still they have not fully addressed the issue of reliable prediction because the observed implied volatility surface is skewed and tends to flatten out for longer maturities; the two models abilities to produce accurate results are reduced. This study ventures into a research that will involve volatility estimation using European logistic-type option pricing with jump diffusion. The knowledge of logistic Brownian motion will be used to develop a logistic Brownian motion with jump diffusion model for price process.

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Keywords: Black-Scholes formula, Brownian motion, Logistic Brownian motion, Jump diffusion, Volatility.

1. Introduction

The supply and demand curves do play a very important role in determining the price and quantity at which assets are bought or sold through an interplay of demand curve and supply curve that causes a market equilibrium. In stock markets the price of an asset is assumed to respond to excess demand and is expressed as;

$$ED(S(t)) = Q_D(S(t)) - Q_S(S(t)), \tag{1}$$

where ED(S(t) is excess demand, $Q_D(S(t))$ are quantities demanded and $Q_S(S(t))$ are quantities supplied at a given time t and price $S(t)$. A standard model for price of stock as a function of time $S(t)$ evolves according to geometric Brownian motion

$$dS(t) = \mu S(t)dt + \sigma S(t)dW(t), \tag{2}$$

* *E-mail: amulambula@kibu.ac.ke*

where μ is the rate of growth of the asset, σ is the volatility and $dW(t)$ is the stochastic function Black-Scholes [2]. This model is based on the idea that prices appear to be the previous price plus some random change and that these price changes are independent, prices being taken to follow some random walk-type behaviour. This is the basis for including the stochastic function. The demand and supply curves are also used to determine the quantity and price at which assets are bought and sold. The supply curve shows what the quantities the sellers are willing and able to sell at various prices whereas demand curve shows the quantities the consumers are willing and able to buy at different prices. The interplay between supply and demand brings in what is called market equilibrium. This is the situation where there is no tendency for change in security price and quantity. In other words, there is no reason for the market price of products to rise or fall. In stock markets the price of an asset is assumed to respond to excess demand as given by Equation (1). The market structure with forces of demand and supply experience upward and downward shifts until a state of market equilibrium is achieved. Stock prices may change due to the general economic factors such as demand and supply, changes in economic outlook and capitalization rates. These brings about small or marginal movements in stock's price hence modeled by a geometric Brownian motion. On the other hand the stock's price may fluctuates due to announcement of some important information causing over-reaction or under-reaction of the asset prices due to good and/or bad news. This information may emanate from the firm or industry. Such information that arrives at discrete points in time can only be modeled by a jump process.

To be able to produce more accurate option pricing, the jump diffusion models were introduced by Merton [5]. The jump diffusion models unlike the famous Black-Scholes models do not make the same assumptions of normally distributed logarithmic returns.

2. Preliminaries

In this section, we look at some fundamental concepts that will be of importance to our study:

2.1. Stochastic process

Any variable whose value changes over time in uncertain way is said to follow a stochastic process. Hence it obeys laws of probability. Algebraically, a stochastic process $X = [X(t); t\epsilon(0, \alpha)]$ is a collection of random variables such that for each t in the index set $(0, \alpha)$, $X(t)$ is a random variable where $X(t)$ is the state of the process at time t. A discrete time stochastic is the one where the value of the variable can only change at a certain fixed points in time. On the other hand continuous time stochastic, change can take any value within a certain range.

2.2. Markov Process

This is a particular type of Stochastic process where only the present value of the variable is relevant for predicting the future. It is believed that the current price already contain what is relevant from the past. It implies that the probability distribution of the price at any particular future time is not dependent on the particular path followed by the price in the past Hull [3]. Stock prices are assumed to follow Markov process.

2.3. Wiener process or Brownian motion

Is a particular type of Markov Stochastic process with a mean change of zero and a variance of 1.0 per year. It follows a stochastic process where μ is the mean of the probability distribution and σ is the standard deviation. That is $W(t) \sim N(\mu, \sigma)$ then for Wiener process $W(t) \sim N(0, 1)$ which means $W(t)$ is a normal distribution with $\mu = 0$ and $\sigma = 1$.

2.4. Generalised Wiener process

The basic Wiener process, dW that has been developed so far has a drift rate of zero and a variance rate of 1.0. The drift rate of zero means that the expected value of W at any future time is equal to its current value. The variance rate of 1.0 means that the variance of the change in W in time interval of length T equals T. A generalised Wiener process for a variable X can be defined in terms of dW as

$$dX = adt + bdW \tag{3}$$

Where mean rate a and variance rate b are constants, adt is the expectation of dX and bdW is the addition of noise or variability to the path followed by X, while b is the diffusivity. In a small interval Δt, the change in the value of $X, \Delta X$ is of the form

$$\Delta X = a\Delta t + b\epsilon\Delta t \tag{4}$$

where as already defined ϵ is a random variable drawing from standardised normal distribution thus the distribution of ΔX is $Mean = E(\Delta X) = a\Delta t$ $Variance(\Delta X) = b^2\Delta t$ thus, Standard deviation of $\Delta X = b\sqrt{\Delta t}$. Hence $\Delta X \sim N(a\Delta t, b\sqrt{\Delta t})$ Similar argument to those given for a Wiener process show that the change in the value of X in any time interval T is normally distributed with mean of change in $X = aT$ Standard deviation of change in $X = bT$, Variance of change in $X = b^2T$ Hence dX $N(aT, b\sqrt{T})$

2.5. It$\widehat{o}$ s Process

This is the generalised Wiener process in which the parameters a and b are functions of the value of the underlying variable X and time t. An It$\widehat{o}$ process can be written algebraically as

$$dX = a(X,t)dt + b(X,t)dW \tag{5}$$

Both the expected drift rate and variance rate of an It$\widehat{o}$ process are liable to change over time. In a small time interval between t and $t + \Delta t$, the changes from X to $X + \Delta X$, is expressed as

$$\Delta X = a(X,t)\Delta t + b(X,t)\epsilon\sqrt{\Delta t} \tag{6}$$

This relationship involves a small approximation. It assumes that the drift and variance rate of X remain constant, equal to $a(X,t)\Delta t$ and $b^2(X,t)\Delta t$ respectively during the interval between t and $t + \Delta t$ hence $\Delta X \sim N(a(X,t)\Delta t, b(X,t)\sqrt{\Delta t})$

2.6. It$\widehat{os}$ Lemma

This is the formula used for solving stochastic differential equations. Suppose that the value of a variable X follows It$\widehat{o}$ s Process

$$dX = a(X,t)dt + b(X,t)dW, \tag{7}$$

where dW is a wiener process and a and b are functions of X and t. The variable X has a drift rate of a and a variance of b^2. It$\widehat{o}$ s Lemma shows that a function $G(X,t)$ twice differentiable in X and once in t, is also an It$\widehat{o}$ process given by

$$dG = \left(\frac{\partial G}{\partial X}a + \frac{\partial G}{\partial t} + \frac{1}{2}\frac{\partial^2 G}{\partial X^2}b^2\right)dt + \frac{\partial G}{\partial X}bdW \tag{8}$$

Where the dW is the same Wiener process, thus G also follows an Itô Process with a drift rate of $\left(\frac{\partial G}{\partial X}a + \frac{\partial G}{\partial t} + \frac{1}{2}\frac{\partial^2 G}{\partial X^2}b^2\right)$ and a variance rate of $\left(\frac{\partial G}{\partial X}\right)^2 b^2$

2.7. Geometric Brownian Motion

A specific Itô Process is the geometric Brownian motion of the form

$$dX = aXdt + bXdW \tag{9}$$

Where $a(X,t) = aX$ and $b(X,t) = bX$. In the above equation geometric Brownian motion has been applied in stock pricing and is given as

$$dS = \mu Sdt + \sigma SdW, \tag{10}$$

where S is the stock price μ is the expected rate of return per unit time and σ is the volatility of the stock price. The equation can be written as;

$$\frac{dS}{S} = \mu dt + \sigma dW \tag{11}$$

This model is the most widely used model of stock price behaviour. A review of this model gives a discrete time model,

$$\frac{\Delta S}{S} = \mu\Delta t + \sigma\epsilon\sqrt{\Delta t}, \tag{12}$$

where ΔS is the change in stock price S within a small interval of time Δt and ϵ is a random variable drawn from standardised normal distribution with mean zero and standard deviation 1. Hence in a short time Δt, the expected value of return is $\mu\Delta t$ and the stochastic component of the return is $\sigma\epsilon\sqrt{\Delta t}$. The variance of the fractional rate of return is $\sigma^2\Delta t$ and $\sigma\sqrt{\Delta t}$ is the standard deviation. Therefore $\frac{\Delta S}{S}$ is normally distributed with mean $\mu\Delta t$ and standard deviation $\sigma\sqrt{\Delta t}$ or $\frac{\Delta S}{S} \sim N(\mu\Delta t, \sigma\sqrt{\Delta t})$

2.8. Dupire's local volatility equation

Before Breeden and Litzenberger (1978), risk-neutral probability density function could be derived from the market prices of European options. Stochastic volatility models posed computational challenges of fitting parameters to the current market prices of options. Researchers and practitioners had to look for a simpler way of pricing exotic options consistent with the volatility smile/skew. Dupire (1994) and Derman and Kani (1994) noted that under risk-neutral there was a unique volatility function $\sigma(S,t)$ consistent with these distribution. For a given current stock price S_0 and a given expiration period T, the collection $C(S_0, K, T)$; $K(0,\infty)$ of discounted option prices of different strikes yields the risk-neutral function ϑ of the final spot price S_T. Suppose the stock prices diffuses with risk-neutral drift $\mu(t)$ and local volatility $\sigma(S,t)$ according to the equation

$$dS(t) = \mu(t)Sdt + \sigma S(t)dW \tag{13}$$

Then,

$$C(S_0, K, T) = \int_K^\infty \vartheta(S_T, T; S_0)(S_T - K)dS_T \tag{14}$$

Where $\vartheta(S_T, T; S_0)$ is the probability density of the final spot price at time T. This pseudo probability density function evolves according to Fokker-Plank equation

$$\frac{\partial \vartheta}{\partial T} = -S\frac{\partial(\mu_{S_T}\vartheta)}{\partial T} + \frac{1}{2}\frac{\partial^2(\sigma^2 S_T^2 \vartheta)}{\partial S_T^2} \tag{15}$$

Differentiating twice with respect to K gives

$$\frac{\partial C}{\partial K} = \int_K^\infty \vartheta(S_T, T; S_0)dS_T \tag{16}$$

$$\frac{\partial^2 C}{\partial K^2} = \vartheta(S_T, T; S_0) \tag{17}$$

Differentiating twice with respect to T we obtain;

$$\frac{\partial C}{\partial T} = \int_K^\infty \frac{\partial}{\partial T}\vartheta(S_T, T; S_0)(S_T - K)dS_T \tag{18}$$

$$\frac{\partial C}{\partial T} = \int_K^\infty \left[\frac{1}{2}\frac{\partial^2}{\partial S_T^2}(\sigma^2 S_T^2 \vartheta) - \frac{\partial(\mu_{S_T}\vartheta)}{\partial T}\right](S_T - K)dS_T \tag{19}$$

Integrating (16) by parts twice gives

$$\frac{\partial C}{\partial T} = \frac{\sigma^2 K^2 \vartheta}{2} + \int_K^\infty \mu_{S_T}\vartheta dS_T \tag{20}$$

$$\frac{\partial C}{\partial T} = \frac{\sigma^2 K^2}{2}\frac{\partial^2 C}{\partial K^2} + \mu(T)C - K\frac{\partial C}{\partial K} \tag{21}$$

Equation (21) is called the Dupire local volatility equation

3. Main results

We begin with deriving Black-Scholes option pricing model..

3.1. Geometric Brownian motion with Jump Diffusion model

Black Scholes [2] approach to option price estimations and option trading brought about a great breakthrough in financial mathematics. In their assumptions, the price of an option follows a geometric Brownian motion. From empirical study, geometrical Brownian motion cannot accurately reflect all behaviours of the stock quotation. Merton who was also involved in the process of developing the Black-Scholes model came up with Merton jump model (1976) as a better estimation of option prices in a precise way. This model where the asset price has jumps superimposed upon a geometric Brownian motion is given by;

$$dS(t) = S(t)(\mu - \lambda k)dt + S(t)\sigma dW + S(t)(q-1)dq(t) \tag{22}$$

where μ is expected return from the asset, λ is the rate at which jumps happen and $k = \varepsilon(q-1)$ is the average jump size measured as a proportional increase in the asset price. $(q-1)$ is the random variable percentage change in the asset price if the poisson event occurs,ε is the expectation operator over the random variable q, $dZ(t)$ is the standard Wiener process and $dq(t)$ is the independent Poisson process generating the jumps. μdt is adjusted by $\lambda k dt$ in the drift term to make the jump part unpredictable innovation Merton [5].

The model consists of two parts; a diffusion component modelled by a Brownian motion describing the instantaneous part of unanticipated return due to normal price vibrations and a jump component modelled by a Poisson process describing the part due to the abnormal price vibrations. The asset price jumps are assumed to be independently and identically distributed. The probability of the Poisson process can be described as

(i). P {the event does not occur in the time interval $(t, t+h)\} = 1 - \lambda\psi h + O(\psi)$.

(ii). P {the event occurs once in the time interval $(t, t+h)\} = \lambda\psi h + O(\psi)$.

(iii). P {the event occurs more than once in the time interval $(t, t+h)\} = O(\psi)$.

Therefore, this can be described as;

$$\frac{dS(t)}{S(t)} = \begin{pmatrix} (\mu - \lambda k) + \sigma dW(t) & \text{if the poisson event does not occur} \\ (\mu - \lambda k) + \sigma dW(t) + q - 1 & \text{if the poisson event does occur} \end{pmatrix}$$

Thus if $\lambda = 0$, also $q - 1 = 0$, then the stock price return is equivalent to Black-Scholes and Merton approaches. Solving for (22) gives;

$$S(t) = S(0)exp[(\mu - \lambda k - \frac{\sigma^2}{2})t + \sigma W(t)] \prod_{i=1}^{i=N(t)} q_i, \tag{23}$$

where $N(t)$ is a poisson process with rate λ, W(t) is a standard Brownian motion and μ is the drift rate. q_i is a sequence of independent identically distributed (i.i.d) non-negative random variables. Merton [5] assumed that $log(q_i) = Y_i$ is the absolute asset price jump size and normally distributed.

3.2. Logistic Brownian Motion-(Non-Linear Brownian Motion)

Non- linear Brownian motion is obtained by introducing excess demand functions applying them in the framework of the Walrasian (Walrasian-Samuelson) price adjustment mechanism Onyango [8]. The core principle of Standard Walrasian model states that asset price changes are directly driven by excess demand for security. For simplicity we do not allow cross-security effects that might be experienced in multi-security market where price of one security reacts to the excess demand of another. The dynamic adjustment rule in such simplified markets may be expressed in continuous-time Walrasian-Samuelson form by a rate of return;

$$\frac{1}{S(t)}\frac{dS(t)}{dt} = kED(S(t)), \tag{24}$$

where the parameters t represents continuous time, and $k > 0$ is a positive market adjustment coefficient (known as speed of market adjustment). $kED(S(t))$ is excess demand taken as continuous function of price $S(t)$. In terms of supply and demand functions $Q_S(S(t))$ and $Q_D(S(t))$, the excess demand is given by

$$ED(S(t)) = Q_D(S(t)) - Q_S(S(t)) \tag{25}$$

The Walrasian-Samuelson adjustment mechanisms of the j^{th} asset is given by

$$\frac{1}{S_j(t)}\frac{dS_j(t)}{dt} = \begin{Bmatrix} h_j ED_j S(t), & j = 1, 2, 3, ..., n \\ 0, & if \quad S_j(t) = 0 \end{Bmatrix}$$

Here $S_j(t)$ is the price of the j^{th} asset, $h_j ED(S(t))$ is the total excess demand function for the j^{th} asset as a function of $S(t)$ for the whole market and h_j is any (fixed) monotonic increasing differentiable real-valued function Samuelson [9]. Approximating h_j by linear function, we have

$$\frac{1}{S_j(t)}\frac{dS_j(t)}{dt} = k_j Q(D_j)(S(t)) - (Q_(S_j)(S(t))), \tag{26}$$

where k_j is a positive adjustment coefficient and interpreted as the "speed of adjustment" of the market changes in supply and demand. In deterministic price adjustment model we can make price adjustment model more computational, we take supply and demand functions to be fixed functions instantaneous price S(t). Then at Walrasian equilibrium price point S^*, $Q_D(S^*) = Q_S(S^*)$, since excess demand is equal to zero. On the other assumption of fixed supply and demand curves, S^* is constant. Away from equilibrium, excess demand for the security will raise its price $S(t)$. And an excess supply will lower its price. Thus the rate of change of price, $S(t)$, with respect to time, t, will depend on the sign of the excess demand. The linear form of $Q_D(S(t))$ and $Q_S(S(t))$ about the constant equilibrium price S^*, gives the deterministic model of price adjustment as

$$\frac{1}{S(t)}\frac{dS(t)}{dt} = k(\alpha + \beta)(S^* - S(t)), \tag{27}$$

where $Q_D(S(t)) = \alpha(S^* - S(t))$, $Q_S(S(t)) = -\beta(S^* - S(t))$ and a constant α and β are demand and supply sensitivities respectively. In logistic Brownian motion we model random fluctuations in supply and demand by changes $\delta\alpha$ and $\delta\beta$ in their respective sensitivities. We consider that $Q_D(S(t))$ and $Q_S(S(t))$ to represent average effects of demand and supply, and suppose that both curves steepen or level off in response to random observed trades, cumulatively they execute a random walk or Wiener diffusion process. From the above models we have $\frac{dS(t)}{dt} = (k(\alpha+\beta) + k(\delta\alpha + \delta\beta))S(t)(S^* - S(t))$ or

$$\frac{dS(t)}{S(t)(S^* - S(t))} = k(\alpha+\beta)dt + k(\delta\alpha + \delta\beta)dt \tag{28}$$

From equation above with $k(\alpha+\beta) = \mu$ (logistic growth parameter) and $k(\delta\alpha + \delta\beta)dt = \sigma dZ$ (noise process) we obtain the simpler form

$$\frac{dS(t)}{S(t)(S^* - S(t))} = \mu dt + \sigma dW \tag{29}$$

Given that trading produces many small random shocks, it is plausible to suppose that $Z = \epsilon\sqrt{t}$, with random number ϵ following a standard normal distribution. Parameter σ is analogous to the price volatility of security trading in the steadier market conditions modelled by dynamical geometric Brownian equation. The equation defines an Itô s Process evolving according to the stochastic differential equation;

$$dS(t) = \mu S(t)(S^* - S(t))dt + \sigma S(t)S^* - S(t)dW \tag{30}$$

We refer to this equation as logistic Brownian motion model or logistic stochastic differential equation. Where $S(t)$ is the price of the underlying asset at any time t, S^* is the market equilibrium, μ is the rate of increase of the asset price, σ is the volatility of the underlying asset and $dZ(t) \sim N(0, dt)$. The solution of the model using It$\widehat{o}$ Lemma is given by

$$\ln|\frac{S(t)}{S^* - S(t)}| = \ln|\frac{S(0)}{S^* - S(0)}| + \mu S^*(t - t_0) + \sigma S^* Z(t) \tag{31}$$

Rearranging and simplifying the above model and solving for S(t) we get

$$S(t) = \frac{S^* S(0)}{S(0) + (S^* - S(0))e^{-(\mu S^*(t-t_0) + \sigma S^* Z(t))}} \tag{32}$$

This price dynamic is referred to us as logistic Brownian motion of stock price S(t) Oduor(2012.

3.3. Logistic Brownian motion with jump diffusion

Using the stochastic differential equation and incorporating the jump diffusion process in Geometric Brownian motion we get

$$\frac{dS(t)}{S(t)(S^* - S(t))} = (\mu - \lambda k)dt + \sigma dW + dq \tag{33}$$

Solving for $S(t)$ we will finally get

$$S(t) = \frac{S^* S(0)}{S(0) + (S^* - S(0))e^{-((\mu-\lambda k)S^*(t-t_0)+\sigma S^* Z(t)+S^* q(t))}} \tag{34}$$

This price dynamic is referred to us as logistic Brownian motion with jump diffusion of stock price $S(t)$, with the initial price $S(0)$, equilibrium price S^*, μ is the expected return from the asset, λ is the rate at which jumps happen and k is the average jump size measured as a proportional increase in asset price and q is the poison process generating jumps.

3.4. Estimation of volatility when asset price is discontinuous

Walrasian price-adjustment model built a non-linear Brownian motion by introducing excess demand. A deterministic logistic equation is obtained by applying excess demand in the framework of Walrasian-Samuelson price adjustment mechanisms. Using the approach of Dupire we derive a diffusion process when the price follows non-linear Brownian motion. Suppose that the price of the asset evolves according to the logistic jump diffusion equation

$$dS(t) = (\mu - \lambda k)S(t)(S^* - S(t))dt + \sigma S(t)S^* - S(t))dW + S(t)S^* - S(t))(q-1)dN \tag{35}$$

Where μ is the growth rate, σ is the volatility, λ is the rate at which the jumps happen, k is the average jump size measured as a proportional increase in asset price, S^* is the equilibrium price is the equilibrium price which is greater than asset price $S(t)$. The aim is to show that there is a unique volatility function $\sigma(S,t)$ such that the observed option price is consistent with equation (35). If we apply the black-Scholes Merton PDE for any claim of asset value $f(S,t)$, we have

$$\frac{\partial f}{\partial t} + \frac{\varphi^2 S^2 \sigma^2}{2}\frac{\partial^2 f}{\partial S^2} + \lambda E[f(qS,t) - f(S,t)] - \lambda\frac{\partial f}{\partial S}S\varphi E(q-1) + rS\varphi\frac{\partial f}{\partial S} - rf = 0 \tag{36}$$

Where $r(t)$ is the risk-free interest rate in the market since we are dealing with price derivatives. If we consider a European call option the process or finding a fair option value of $f(S,t)$, will depend on asset price $S(t)$ and time t. Therefore the function $f(S,t)$ can be written for the value of the contract with boundary condition

$$f(S,t) = \max(S^* - S(t), 0) \tag{37}$$

At a time t before expiry date the price of the call option will be a function of $S(t), t, T$,and S^* that is $f(S(t), t, T, S^*)$. When we fix the expiry rate T and the equilibrium S^* we have

$$f(S,t) = C(t,T)\int_S^\infty \max(S^* - S(t), 0)\vartheta dS(T) \tag{38}$$

Differentiating (38) twice with respect to K we get;

$$\vartheta = K = \frac{\partial^2 f(S,t)}{\partial S^2} \tag{39}$$

Applying the Fokker-Plank equation and using Kolmogorov's foward equation on (35) we obtain

$$\frac{\partial\vartheta}{\partial T} - \frac{1}{2}\frac{\partial^2 f(S,t)}{\partial S(T)^2}\sigma^2 S^2\varphi^2\vartheta + \lambda E[f(qS,t) - f(s,t)]\vartheta - \frac{\partial f(S,t)}{\partial S(T)}S\varphi\vartheta E(q-1) + \frac{\partial f(S,t)}{\partial S(T)}r\varphi S\vartheta = 0 \tag{40}$$

$$\frac{\partial f}{\partial T} - \frac{1}{2}\frac{\partial^2 f(S,t)}{\partial K^2}\sigma^2 S^2\varphi^2 + \lambda E[f(qS,t) - f(s,t)] - \frac{\partial f(S,t)}{\partial K}K\varphi E(q-1) + \frac{\partial f(S,t)}{\partial K}r\varphi K = 0 \tag{41}$$

$$\sigma^2 = 2\left\{\frac{\frac{\partial f}{\partial T} + \lambda E[f(qS,t) - f(S,t)] - \frac{\partial f}{\partial K}K\varphi E(q-1) + \frac{\partial f}{\partial K}r\varphi K}{K^2\varphi^2\frac{\partial^2 f}{\partial K^2}}\right\}, \tag{42}$$

4. Conclusion

In this paper we have developed European logistic-type option pricing with jump diffusion model. The results obtained are useful to investors and practitioners in estimation of volatilty which is an important tool in options and assets trading.

References

[1] R. Apaka, S. N. Onyango, O. Omolo and O. Nyakinda, *Modelling of Jumps and Mean Reversion in Kenya Interest rates*, International Journal of marketing and Technology, 5(11)(2015), 8-11.

[2] F. Black and N. Scholes, *The pricing options and corporate liabilities*, Journal of political Economy, 18(1973), 637-659.

[3] C. J. Hull, *Options, Futures and other derivatives*, Joseph L.Rotman School of Management, University of Toronto 4th Edition, (2000).

[4] S. G. Kou, *A Jump-Diffusion Model for Option Pricing*, Management Science, 48(8)(2002), 1086-1101.

[5] C. R. Merton, *Option Pricing When Underlying Stock Returns are Discontinuous*, Masachusetts Institute of Technology, 16(22)(1975), 225-443.

[6] O. J. Nyakinda, *Derivation of Non-Linear Logistic Black-Scholes-Merton Partial Differential Equation*, PhD Thesis, Jaramogi Oginga Odinga University of Science and Technology, (2011).

[7] D. B. Oduor, O. N. Omolo, N. B. Okelo and S. N. Onyango, *Estimation of Market Volatility: A case of logistic Brownian motion*, International Journal of Marketing and Technology, 2(1)(2012), 32-48.

[8] S. N. Onyango, *Extracting Stochastic process from market price data: A pattern recognition approach*, PhD Thesis, University of Huddersfield, (2003).

[9] P. A. Samuelson, *The stability of equilibrium*, Comparative Statistics and Dynamics, (Econometrics), 9(1965), 97-120.

[10] P. A. Verlhust, *Notice on the law that the population follows in its increase*, Mathematical and Physical Correspondence, 10(1838), 113-121.