**JARAMOGI OGINGA ODINGA UNIVERSITY OF**

**SCIENCE AND TECHNOLOGY**

**SCHOOL OF MATHEMATICS AND STATISTICS**

**UNIVERSITY EXAMINATION FOR DEGREE OF BACHELOR OF SCIENCE IN ACTUARIAL SCIENCE WITH IT**

**2$^{ND}$ YEAR 2$^{ND}$ SEMESTER 2019/2020 ACADEMIC YEAR**

**MAIN (RESIT)**

---

**COURSE CODE: SAC 206**

**COURSE TITLE: ACTUARIAL MATHEMATICS I**

**EXAM VENUE:** **STREAM: (BSc)**

**DATE:** **EXAM SESSION:**

**TIME: 2.00 HOURS**

---

**Instructions:**

1. **Answer question 1 (Compulsory) and ANY other 2 questions**
2. **Candidates are advised not to write on the question paper.**
3. **Candidates must hand in their answer booklets to the invigilator while in the examination room.**

QUESTION ONE

(a) Define the following terms as used in actuarial mathematics;

i. Contingent probability.

ii. ${}_tp_{xy}$

iii. ${}_tp_{\overline{xy}}$ [5 marks]

(b) In a certain population where a third are females and two-thirds are males, the force of mortality for a female newborn is

$$\mu_x^F = \frac{1}{2}x, \text{ for } x \geq 0$$

and that for a male newborn is

$$\mu_x^M = \frac{8}{9}x, \text{ for } x \geq 0$$

. For a newly selected member of this population, calculate ${}_{1|2}q_3$ and interpret this probability. [5 marks]

(c) Consider the following function for a newborn

$$S_0(x) = \frac{1}{c}(110 - x)^{\frac{2}{3}}, \; 0 \leq x \leq 1000$$

i. Calculate c so that this survival function is legitimate nd give the limiting age for this model. [3 marks]

ii. Calculate the probability that a newborn will reach to age 65 but die within 20 years following that. [2 marks]

iii. Calculate the expected future lifetime of a newborn. [2 marks]

(d) Given the following ${}_5p_{50} = 0.9$, ${}_{10}p_{50} = 0.8$ and $q_{55} = 0.03$. Find the probability that (56) will die within four years. [5 marks]

(e) Joe plans on going to Call Technologies. He will need to pay Kshs. 50000 each when he goes there. In order to do this, he will need to deposit X into an account every month that earns an interest of 8% convertible monthly for seven years. He will take out the payments at the end of the four years at the end of the year. After the last withdrawal, the amount in the account will be exhausted. Calculate X. [5 marks]

## QUESTION TWO

(a) Let X be an age at death random variable. If mortality is described by

$$s(x) = (1 - \frac{x^2}{8100}) \qquad \text{for } 0 \leq x \leq 90$$

Determine

i. $\mathring{e}_0$ and interpret this value.

ii. The probability that a life age 35 dies before age 55 years.

iii. $\mu(40)$ [5 marks]

(b) Mortality of Audra, age 25, follows a DeMoivre's law with age as 100. If she takes up a hot air ballooning for the coming year, her assumed mortality will be adjusted so that for the coming year only, she will have a constant mortality of 0.1. Calculate the increase in the 11- year temporary complete life expectancy for Audra if she takes up hot air ballooning. [5 marks]

(c) For an age at death random variable, X, K is the curtate future lifetime random variable. Given

$$q_{x+k} = 0.1(k+1) \qquad \text{for } k = 0.1, 2, \cdots, 9$$

. Calculate $Pr(K = 3)$ [5 marks]

(d) Suppose you are given that

$$l_x = 1000(27 - \frac{3}{10}x)^{\frac{1}{3}} \qquad \text{for } 0 \leq x \leq 90.$$

Calculate the future average lifetime for an individual. [5 marks]

## QUESTION THREE

(a) Suppose you are given the survival function

$$s(x) = (1 - \frac{x}{\omega})^{\alpha} \qquad \text{for } 0 \leq x \leq \omega$$

. Find $\mu_x \times \mathring{e}_x$. (show your working) [5 marks]

(b) Given that $\mu_x = F + e^x$ and ${}_{0.4}p_0 = 0.5$. Calculate the value of $F$. [3 marks]

(c) Show that ${}_tp_x = 1 - t \cdot q_x$ under uniform distribution of deaths assumption. [3 marks]

(d) For a given life age 30, it is estimated that an impact of a medical breakthrough will be an increase of 4 years on $\mathring{e}_{30}$. Prior to the medical breakthrough, the survival function follows a DeMoivre's law with omega as 100. Assuming that the DeMoivre.s law still applies after the medical breakthrough, calculate the new omega. [6 marks]

(e) Let X be the age at death random variable. Assume that $X \sim$ DeMoivre$^0$s law with omega as 100. Calculate the $\mu_{30}$. [3 marks]

QUESTION FOUR

(a) Mat takes out a loan for a car Kshs. 35000. He must make 16 annual payments of Ksh. 4000. For the first seven years, the interest is 8%, what is the annual effective rate of interest for the last 9 years. [5 marks]

(b) Calculate the value of ${}_{1.75}p_{45.5}$ on the basis of Illustrative Life Table and assuming that deaths are uniformly distributed between integral ages. [5 marks]

(c) A perpetuity immediate has annual payments. The first payment is 1 and each subsequent payment increases by 1 until the payment reaches 20. The payments stay level thereafter. Find the present value of the perpetuity at an annual effective interest rate of 6%. [5 marks]

(d) Suppose for an 80 year old fellow, his force of mortality is given by

$$\mu_{80+t} = \frac{1}{10 - t} \qquad \text{for } 0 \leq t \leq 10.$$

Calculate the probability that this fellow will die between ages 85 and 90. [6 marks]

QUESTION FIVE

(a) Calculate the value of an $I(a)_{50}$ assuming an AM92 mortality and 4% p.a interest. [5 marks]

(b) A population is subject to a constant force of mortality of 0.015, calculate

i. The probability that a life age 20 exact will die before age 21.25 exact. [3 marks]

ii. The curtate expectation of life age 20 exact. [2 marks]

(c) You are given

i. $q_{50} = 0.3$ and $q_{61} = 0.4$.

ii. $f$ is the probability that 60.5 will die between ages 60.5 and 61.5 under UDD.

iii. g is the probability that 60.5 will die between ages 60.5 and 61.5 under the Balducci assumption.

Calculate $10000(g - f)$. [5 marks]

(d) For a special fully continuous last survivor insurance of 1 on two independent lives (x) and (y), you are given

i. Death benefits are payable at the moment of second death.

ii. Level benefit premiums, are payable only when (x) is alive and (y) is dead; no premiums are payable while both are alive or of (x) dies first.

iii. $\sigma = 0.5$, $\mu_x(t) = 0.03$, $t \geq 0$, $\mu_y(t) = 0.04$, $t \geq 0$

Calculate $1000\pi$. [5 marks]