JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY

SCHOOL OF MATHEMATICS AND ACTUARIAL SCIENCE

UNIVERSITY EXAMINATION FOR DEGREE OF BACHELOR OF SCIENCE

IN ACTUARIAL SCIENCE

SPECIAL RESIST 2020/2021 ACADEMIC YEAR

REGULAR

COURSE CODE: SAS 102

COURSE TITLE: PROBABILITY AND DISTRIBUTION THEORY I

EXAM VENUE: **STREAM: BSC. ACTUARIAL SCIENCE**

DATE: **EXAM SESSION:**

TIME: 2.00 HOURS

Instructions:

1. **Answer Question ONE and ANY other two questions**
2. **Candidates are advised not to write on the question paper.**
3. **Candidates must hand in their answer booklets to the invigilator while in the examination room.**

QUESTION ONE (COMPULSORY) – (30 MARKS)

a) The random variables X and Y have joint p.d.f given by

$$f(x,y) = \begin{cases} Cxy, & 0 < x < 1, 0 < y < x \\ 0, & otherwise \end{cases}$$

Compute the value of C hence $E(y)$ (6 marks)

b) The random variables X and Y have joint discrete distribution given by

$$f(xy) = \begin{cases} \frac{x+2y}{18}, & x = 1,2 ; y = 1,2 \\ 0, & otherwise \end{cases}$$

Find $E(X/Y)$ (6 marks)

c) The relative humidity Y when measured at a given location has a probability density function given by

$$f(y) = \begin{cases} Ky^3(1-y)^2, & 0 < y < 1 \\ 0, & otherwise \end{cases}$$

Find K given this is a Beta density function hence the probability that the proportion of humidity is better than 50%. (6 marks)

d) The weekly amount of shut down X for a manufacturing plant has approximately a gamma distribution with $\alpha = 5$ and $\beta = 3$. The loss to the company in thousands of shillings due to shut down is given by

$$L = 100 + 40x + 200x^3$$

Find the expected loss due to a single shut down. (6 marks)

e) Assume that X is normally distributed with a mean of 6 and a standard deviation 4. Determine (6 marks)

i. $P(X > 0)$
ii. $P(3 < X < 7)$
iii. $P(-2 < X < 9)$

QUESTION TWO (20 MARKS)

a) The gamma distribution takes the form $f(x) = \begin{cases} \frac{x^{\alpha-1}e^{-x/\beta}}{\beta^\alpha \Gamma(\alpha)}, & x > 0, \beta > 0, \alpha > 0 \\ 0, & x \leq 0 \end{cases}$, with scale parameter β and shape parameter . Obtain expressions for the mean and variance of tis distribution. (10marks)

b) The joint density function for two random variables X and Y is given by

$$f(x,y) = \begin{cases} k(2x+y), & 0 < x < 3, 0 < y < 5 \\ 0, & otherwise \end{cases}$$

Obtain

i. the value of k (4marks)
ii. The marginal distributions of X and Y (6marks)

QUESTION THREE (20 MARKS)

a) Given that X assumes the Beta distribution,

$$f(x) = \begin{cases} \frac{\Gamma(\alpha+\beta)}{\Gamma\alpha\Gamma\beta} x^{\alpha-1}(1-x)^{\beta-1}, & 0 < x < 1, \alpha > 0, \beta > 0 \\ 0, & otherwise \end{cases}$$

With $E(X) = \frac{1}{10}$ and $(X^2) = \frac{1}{70}$, obtain expressions for the $E(X)$ and $E(X^2)$ hence the numerical values of α *and* β. (12 marks)

b) Given $f(x,y) = \begin{cases} \frac{1}{9}(xy), & 0 < x < 2, 0 < y < 3 \\ 0, & otherwise \end{cases}$ obtain $var\ (Y/X = x)$ (8marks)

QUESTION FOUR (20 MARKS)

a) The probability density function of a random variable X is given by $f(x) = \begin{cases} 5x^4, & 0 < x < 1 \\ 0, & otherwise \end{cases}$

Determine:

i. The pdf of a random variable $Y = X^3$ (5 marks)

ii. $p(\frac{1}{8} < Y < 1$ (5marks)

b) The joint p.d.f of two random variables X and Y is defined as follows

$$f(x,y) = \begin{cases} \frac{1}{16} xy, & 0 < x < a, 0 < y < b \\ 0, & otherwise \end{cases}$$

Suppose we know that: $E(xy) = 32/9$, $E(x) = 4/3$, and that the variables are independent, determine:

i) The mean of Y.

ii) The values of a and b (10 marks)

QUESTION FIVE (20 MARKS)

a) Determine the value of c for which the function below is a joint probability density function hence compute $cov(XY)$

$$f(x,y) = \begin{cases} c(x+y), & 0 < x < 3, x < y < x+2 \\ 0, & otherwise \end{cases}$$

(10marks)

b) Suppose a random variable X has the uniform distribution in the interval; $-\alpha \leq x \leq \alpha$, where $\alpha > 0$. Determine the value of α such that

i) $P(X > 1) = 1/3$,

ii) $P(X < 0.5) = 3/5$,

iii) Var (X) (10 marks)