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YEAR THREE SEMESTER TWO EXAMINATION (Special Resit)2020

SMA 303: COMPLEX ANALYSIS

INSTRUCTION: Answer Question ONE and ANY other TWO questions.

QUESTION ONE (COMPULSORY) – 30 MARKS

a) Define each of the following terms as used in complex analysis
   i) Disk
   ii) Deleted neighbourhood
   iii) Argument
   iv) Limits of a complex function (8 marks)

b) Express $2-2\sqrt{3}i$ in polar form using the principal argument. (2 marks)

c) Evaluate the integral $\oint_c \frac{z}{z^2+16}dz$, where C is the circle $|z-2i|=4$ using the Cauchy's integral formular. (4 marks)

d) Compute the n^th^ root for the $\left(2\sqrt{3}+i\right)^{\frac{1}{2}}$, hence sketch an appropriate circle indicating the roots w_0 and w_1,. (4 marks)

e) Sketch the set S denoted by the inequality $2 \leq |z-3+i| < 3$. (4 marks)

f) Find the image of a line $x = 1$ under the complex mapping $w=z^2$ for $w,z \in \mathbb{C}$, hence sketch the line and its image under the mapping (4 marks)

g) Evaluate the line integral $I=\oint_c \left(x^2dx-2ydy\right)$ where C comprises the triangle $O(0,0)$, $A(2,1)$ and $C(1,3)$ (4 marks)

QUESTION TWO (20 MARKS)

a) Prove that if a complex function $f(z)=u(x,y)+iv(x,y)$ is analytic at any point z, and in the domain D,then the Laplace's equation $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=0$, can be verified. (7 marks)

b) Find the derivative of $\frac{z^2-2iz}{2z+4-i}$ (3 marks)

c) Solve for w, given the complex function $e^{w} = \sqrt{3} + i$ for $w, \in \mathbb{C}$. (6 marks)

d) Compute the principal value of the complex logarithm $\ln z$ for $z = 1 - i$ (4 marks)

QUESTION THREE (20 MARKS)

a) State De-Moivre's theorem hence use it to evaluate $(1-i)^6$, giving your answer in the form $a + bi,\ a, b \in \mathbb{R}$ (7 marks)

b) Find an upper bound for the reciprocal of $z^4 - 5z + 1$, given that $|z| = 2$. (5 marks)

c) Use the definition of the derivative of a complex function to determine the derivative of $f(z) = \dfrac{1}{z}$ in the region where the derivative exists. (5 marks)

d) Evaluate $\left(\dfrac{2+i}{\sqrt{3}+i}\right)^{\frac{1}{4}}$, giving all your answers in polar form. (6 marks)

QUESTION FOUR (20 MARKS)

a) Find the value of i^i (4 marks)

b) Given that $e^{i\theta} = \cos\theta + i\sin\theta$ for any real number θ, prove that $e^{iz} = \cos z + i\sin z$ for any complex number z. (6 marks)

c) Evaluate $\oint \dfrac{1}{z}\,dz$, where $C$is the circle $x = \cos t\,,\ x = \sin t$ for $0 \le t \le 2\pi$ (4 marks)

d) State the Cauchy's integral formular for derivatives hence evaluate

$$\oint \frac{z^2 + 3}{z(z-i)^2}$$

(6 marks)

QUESTION FIVE (20 MARKS)

a) Find the real numbers p and q for which the complex numbers $z = a + bi$ and $w = a + \frac{1}{b}i$ are equal given that $w, z \in \mathbb{C}$. (3 marks)

b) Show that the function $f(z) = 3x^2y^2 - 6ix^2y^2$ is not analytic at any point but differentiable along the coordinate axes. (6 marks)

c) Use L'Hopital's rule to compute

$$\lim_{z \to 1+i} \frac{z^5 + 4z}{z^2 - 2z + 2}$$

(5 marks)

d) Given the complex function $f(z) = u(x, y) + iv(x, y)$, verify that the function $u(x, y) = 2x - 2xy$, hence find $v(x, y)$ the harmonic conjugate u,Hence find the corresponding analytic function $f(z) = u + iv$. (6 marks)