



**JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY**

**SCHOOL OF MATHEMATICS AND ACTUARIAL SCIENCE**

**UNIVERSITY EXAMINATION FOR DEGREE OF BACHELOR OF EDUCATION AND  
ACTUARIAL SCIENCE**

**2<sup>ND</sup> YEAR 1<sup>ST</sup> SEMESTER 2022/2023 ACADEMIC YEAR**

**COURSE CODE: WAB 2203**

**COURSE TITLE: FUNDAMENTALS OF ACTUARIAL MATHEMATICS II**

**EXAM VENUE:**

**STREAM: EDUCATION, ACTUARIAL**

**DATE:**

**EXAM SESSION:**

**TIME: 2.00 HOURS**

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**Instructions:**

- 1. Answer question one (compulsory) and any other two questions.**
- 2. Candidates are advised not to write on the question paper.**
- 3. Candidates must hand in their answer booklets to the invigilator while in the examination room.**

**Question 1 [30 marks]**

- a. Define i.) whole life assurance [4Marks]  
ii.) term assurance  
iii.) deferred assurance  
iv.) endowment assurance
- b. Show that  $\ddot{a}_x = \sum_{t=0}^{\infty} v^t {}_tP_x$  [4Marks]
- c. Give that  $Z = g(T) = Sv^T$  is the present value of S due immediately on the death of (x), and that the mean of Z is  $S\bar{A}_x$ . What is the variance of Z? [5marks]

- d. Show that [5marks]

$$\bar{a}_x = \frac{\bar{N}_x}{D_x}$$

- e. Write the present value of the benefits for an endowment life assurance payable immediately on death [2marks]
- f. Show that  $A_x = vq_x + vp_x A_{x+1}$  [3marks]
- g. Find the M.P.V. of a whole life assurance of £10,000, payable immediately on the death of (40), according to E.L.T. No. 12-Males with interest at 4% p.a [2marks]
- h. A life aged 40 effects a 20-year without profits endowment assurance policy with a sum assured of sh. 20,000 (payable at the end of the year of death or on survival to the end of the term). Level premiums are payable annually in advance throughout the term of the policy or until earlier death of the life assured. Calculate the level premium,  $P$ , using the following premium basis [5marks]  
Mortality: A1967-70 Ultimate;  
Interest: 6% p.a.  
Expenses: none

**Question 2 [20 marks]**

- a. On the basis of A1967-70 select mortality and 4% p.a. interest, calculate the mean present value of each of the following assurance benefits for a life aged 30:

- (i) A whole life assurance for £10,000, payable immediately on death; [2marks]
- (ii) A 20-year term assurance for £50,000, payable at the end of the year of death; [3marks]
- (iii) A 20-year endowment assurance for £50,000, with the death benefit payable immediately on death; [3marks]
- (iv) A deferred temporary assurance for £100,000, payable at the end of the year of death, if death occurs between ages 40 and 50 exactly. [3marks]

[Use the factor  $(1+i)^{-1}$  for accelerating payments from end of year of death to the moment of death.]

b. Ten years ago life office issued a 20-year endowment assurance without profits to (35). The sum assured is £10,000, payable at the end of the year of death (or on survival for 20 years), and premiums are payable annually in advance. The basis for premiums and reserves is:

A1967-70 ultimate;

6% p.a. interest; expenses are 3% of all office premiums (including the first) with additional initial expenses of

1.5% of the sum assured.

Calculate

(i) the annual premium, and [4marks]

(ii) the reserve,

(a) just before receipt of the premium now due, and [3marks]

b) just after receipt of the premium now due. [2marks]

### **Question 3 [20 marks]**

a. Using commutation functions or otherwise calculate the values of the following: [10Marks]

(i)  $A_{[40]:\overline{10}|}$  on A1967-70, 4% p.a. interest;

(ii)  $A_{30:\overline{20}|}^1$  on A1967-70 Ultimate, 4%;

(iii)  $\bar{A}_{30:\overline{20}|}^1$  on A1967-70 Ultimate, 4%;

(iv)  $\bar{A}_{30:\overline{20}|}$  on A1967-70 Ultimate, 4%;

(v)  $\bar{A}_{30:\overline{20}|}$  on English Life Table No.12 Males, 4%.

b. The staff of a large company is maintained as a stationary population by 500 new entrants each year at exact age 20. One third of those reaching age 30 leave immediately. Of the remainder,  $\frac{1}{4}$  of those attaining age 60 retire immediately and the survivors retire at age 65.

The only other decrement is death.

Calculate

(i) the number of staff, [4marks]

(ii) the number of deaths in service each year. [6marks]

Basis: English Life Table No. 12 – Males

### **Question 4 [20 marks]**

a. A life aged 50 who is subject to the mortality of the A1967-70 Select table, effects a pure endowment policy with a term of 20 years for a sum assured of \$10,000.

i. Write down the present value of the benefits under the contract, regarded as a random variable. [4marks]

ii. Assuming an effective rate of interest of 5% per annum, calculate the mean and the variance of the present value of the benefits available under this contract [16marks]

**Question 5 [20 marks]**

- a. A life office issues a large block of 3-year unit-linked endowment assurances under which 70% of the first year's premium and 102% of subsequent premiums are invested in units at the offer price. The bid price of the units is 90% of the offer price. The units are subject to an annual management charge of 0.85% of the bid value of the fund at the end of each policy year. The annual premium is £2,000 and unit prices are assumed to grow at 8% per annum. Calculate the bid value of the units at the end of each year, according to the office's projections. [14marks]
- b. A man aged 50 has just retired because of ill health. Up to exact age 58 he will be subject to a constant force of mortality of 0.019803 p.a., after which his mortality will be that of E.L.T. No. 12 - Males. Find the probability that he will [6marks]
- (i) die before age 55,
  - (ii) live to age 65,
  - (iii) die between ages 55 and 60.