**JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY**

**SCHOOL OF MATHEMATICS AND ACTUARIAL SCIENCE**

**UNIVERSITY EXAMINATION FOR DEGREE OF BACHELOR OF EDUCATION AND ACTUARIAL SCIENCE**

**3rd YEAR 1st SEMESTER 2022/2023 ACADEMIC YEAR**

**MAIN CAMPUS**

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**COURSE CODE: WAB 2306**

**COURSE TITLE: ACTUARIAL LIFE CONTINGENCIES I**

**EXAM VENUE:** **STREAM: ACTUARIAL SCIENCE**

**DATE:** **EXAM SESSION:**

**TIME: 2.00 HOURS**

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**Instructions:**

1. **Answer question one (compulsory) and any other two questions.**
2. **Candidates are advised not to write on the question paper.**
3. **Candidates must hand in their answer booklets to the invigilator while in the examination room.**

## Question 1 [30marks]

a. With reference to sickness functions, define the following: [5marks]
   i. Deferred period
   ii. Waiting period
   iii. The off-period
   iv. $\bar{z}_x^{m/n}$
   v. $\bar{z}_x^{13}$

b. Define $tP_{xy}$ and show that $tP_{xy} = tP_x + tP_y - tP_{xy}$ [4marks]

c. Prove that: $A_{xy} = 1 - d\ddot{a}_{xy}$ [3marks]

d. You are using the three-state illness-death model to price various sickness policies. Using the notation $P_x^{aa}$, $P_x^{ai}$, $P_x^{ii}$, write down an expression for the expected present value of each of the following sickness benefits for a healthy life aged 30.

i. £3,000 *pa* payable continuously while ill, but ceasing at age 60. [1marks]

ii. £3,000 *pa* payable continuously throughout the first period of illness only, but ceasing at age 60. [2marks]

iii. £3,000 *pa* payable continuously while ill provided that the life has been ill for at least one year. Again, any benefit ceases to be paid at age 60. [2marks]

e. Under U.D. of D. of each mode of decrement in its single-decrement table

$aq_x^{\alpha} = q_x^{\alpha}[1 - {}^1/_2\left(q_x^{\beta} + q_x^{\gamma}\right) + {}^1/_3\, q_x^{\beta}.q_x^{\gamma}]$ . Prove [4marks]

f. Prove that:

$tP_{xy} = \exp(-\int_0^t \mu_{x+r:y+r}\, dr)$ [5marks]

g. Evaluate $\bar{A}_{\overline{40:40}}$ on the basis of *A*1967 - 70 ultimate at 4% [4marks]

## Question 2 [20marks]

A life insurance company issues a joint-life annuity to a male, aged 68, and a female, aged 65. The annuity of £10,000 *pa* is payable annually in arrears and continues until both lives have died. The insurance company values this benefit using PA92C20 mortality (males or females as appropriate) and 4% *pa* interest.

(i) (a) Calculate the expected present value of this annuity. [7marks]

(b) Derive an expression for the variance of the present value of this annuity in terms of appropriate single and joint-life assurance functions. [6marks]

(ii) If the insurance company charges a premium of £150,000 for this policy, calculate the probability that it makes a profit on the contract. [7marks]

## Question 3 [20marks]

a. A friendly society issued a policy providing the following benefits to a man aged exactly 25 at entry:

i. On death at any time before age 60, the sum of £4,000 payable immediately;

ii. On survival to age 60, an annuity of £8 per week payable weekly in advance for as long as he survives;

iii. On sickness, an income benefit to be payable during sickness of £32 per week for the first 6 months reducing to £16 per week for the next 18 months and to £8 per week thereafter.

Sickness benefit is not payable after age 60. There is no waiting period. Premiums are payable monthly in advance for at most 35 years, and are not waived during periods of sickness.

The society uses the following basis to calculate premiums. Find the monthly premium.

mortality: English Life Table No.12 - Males
sickness: Manchester Unity Sickness Experience 1893-97, Occupation Group AHJ
interest: 4% p.a.
expenses: none [10 marks]

b. A life office is proposing to issue 3-year sickness benefit policies to lives aged 30. The benefits are £ 50 per week during sickness within the next three years. There is no waiting period and the off-period is as in the Tables provided. Find the single premium on each of the following bases:
mortality: English Life Table No.12 - Males
interest: (i) 4% p.a., [2marks]
(ii) 5% p.a. [2marks]
sickness: Manchester Unity 1893-97 (AHJ)
expenses: none.

c. Define the following symbols in words, and give a formula in terms of an integral for each of them: [6marks]

(a) $\overline{A}^{\,1}_{xy}$

(b) $\overline{A}_{\overline{xy}}^{\,2}$

(c) $\overline{a}_{y|x}$

## Question 4 [20marks]

The members of a large company's manual workforce are subject to three modes of decrement, death, withdrawal and promotion to supervisor. It is known that these workers' independent rates of mortality are those of English Life Table No. 12 - Males, the independent withdrawal rate is 0.04 at each age, and their independent promotion rate is 0.02 at age 50 and 0.03 at age 51.

a. Draw up a service table for manual workers from age 50 to age 51 with a radix of 100,000 at age 50, including the value of $(al)_{52}$ [18marks]

b. Calculate the probability that a life aged exactly 50 will gain promotion within 2 years. [2marks]

## Question 5 [20marks]

a) A life insurance company uses the three-state illness-death model as described above to calculate premiums for a 3-year sickness policy issued to healthy policyholders aged 60.

$S_t$ denotes the state occupied by the policyholder at age $60 + t$, so that $S_0 = a$ and $S_t = a, i \text{ or } d$ for $t$ = 1, 2, 3.

The transition probabilities used by the insurer are defined in the following way:

$$P_{60+t}^{jk} = P(S_{t+1} = k / S_t = j)$$

For $t$ = 0, 1, 2 , it is assumed that:

$$P_{60+t}^{aa} = 0.9, \quad P_{60+t}^{ai} = 0.08, \quad P_{60+t}^{ia} = 0.7, \quad P_{60+t}^{ii} = 0.25$$

What is the probability that a new policyholder is:

(i) ill at exact age 62? [5marks]

(ii) dead at exact age 62? [5marks]

b) A population is subject to two modes of decrement, $\alpha \text{ and } \beta$ . In the single decrement tables:

${}_tP_{60}^{\alpha} = (\frac{40-t}{40})^2$ for $0 \leq t \leq 40$

and

${}_tP_{60}^{\beta} = (\frac{40-t}{40})^2$ for $0 \leq t \leq 40$

Calculate the value of $(aq)_{60}^{\alpha}$ [10marks]