JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY

SCHOOL OF MATHEMATICS AND ACTUARIAL SCIENCE

UNIVERSITY EXAMINATION FOR DEGREE OF BACHELOR OF SCIENCE ACTUARIAL

4th YEAR 2nd SEMESTER 2021/2022

REGULAR (MAIN)

COURSE CODE: WAB 2404

COURSE TITLE: COMPUTATIONAL FINANCE.

EXAM VENUE: **STREAM: (BSc Actuarial Science)**

DATE: **EXAM SESSION:**

TIME: 2.00 HOURS

Instructions:

1. **Answer question 1 (Compulsory) and ANY other 2 questions**
2. **Candidates are advised not to write on the question paper.**
3. **Candidates must hand in their answer booklets to the invigilator while in the examination room.**

QUESTION ONE

a. Define
   i.) Intrinsic value. (1mark)
   ii.) Write down the intrinsic value of a put option at time t. (2marks)

b. Suppose that the price of Share X is 112 and that a put option on Share X with an exercise price of 110 is currently priced at 5. Calculate the intrinsic value and time value of the option. (2marks)

c. Given a filtered probability space (Ω,F,F_t,P), what are the conditions for a stochastic process X_t to be called a martingale with respect to the filtration, F_t? (3marks)

d. What is meant by saying that the process $\{Y_t\}$ is a martingale with respect to another process $\{X_t\}$? (2marks)

e. Find the stochastic differential equation for $W_t^{2.}$ (2marks)

f. Suppose that the current time corresponds to *t = 5* and that the force of interest has been a constant 4% pa over the last 5 years. Suppose also that the force of interest implied by current market prices is a constant 4% pa for the next 2 years and a constant 6% pa thereafter. If *T=10* and *S=15* , write down or calculate each of the four quantities *P(t,T)* , *r (t)* , *f(t,T,S)* and *r(t,T)* using the notation above. (5 marks)

g. state and explain two basic types of options. (4marks)

h. A fixed-interest security pays coupons of 8% pa half-yearly in arrear and is redeemable at 110%. Two months before the next coupon is due, an investor negotiates a forward contract to buy £60,000 nominal of the security in six months' time. The current price of the security is £80.40 per £100 nominal and the risk-free force of interest is 5% pa. Calculate the forward price. (4 marks)

i. State and explain the assumptions underlying the Black-Scholes model. (5marks)

QUESTION TWO

a. State five parameters used to value an option on a non-dividend-paying share. (5marks)

b. $\{X_t\}$ be a continuous-time stochastic process defined by the equation
$X_t = \alpha W_t^2 + \beta$, where
$\{W_t\}$ is a standard Brownian motion and α β and are constants.
By applying Ito's Lemma, or otherwise, write down the stochastic differential equation satisfied by X_t . (5marks)

c. Assume that the spot rate of interest at time t ,S(t), can be modelled by $S(t)=e^{-2\mu W(t)}$ where
W(t) is a Brownian motion with drift coefficient $\mu$and volatility coefficient 1 such that W(0) = 0.
   (i) Write down an expression for W(t) in terms of a standard Brownian motion, B(t). (2marks)
   (ii) Show that $\{S(t): t>0\}$ is a continuous-time martingale. (8mark)

QUESTION THREE

a. Consider an American put option on a non-dividend-paying share.
List the five factors that determine the price of this option and, for each factor, state whether an increase in its value produces an increase or a decrease in the value of the option. (5marks)

b. Let X be an Ito process that satisfies
$dX_t=\mu(X_t,t)dt +\sigma(X_t,t)dB_t$
whereB_t is a standard Brownian motion. Let $f(X_t,t)$ be a function of t and X_t .
   (i) By considering Taylor's theorem, suggest a partial differential equation that must be satisfied by$f(X_t,t)$ in order that it is a martingale. (3marks)
   (ii) Verify that your equation holds when$f(X_t,t) = B_t^2-t$. (2marks)

c. A process X_t satisfies the stochastic differential equation:
$dX_t = \sigma(X_t)dB_t+ \mu(X_t)dt$
whereB_t is a standard Brownian motion.
Deduce the stochastic differential equation for the process X_t^3. (10marks)

QUESTION FOUR

a. Derive the following relationship. (5 marks)

$$f(0,t,T) = \frac{1}{T-t} log \frac{P(0,t)}{P(0,T} \ for \ t < T$$

b. Under one particular term structure model:

$$f(t,T) = 0.03e^{-0.1(T-t)} + 0.06\left(1 - e^{-0.1(T-T)}\right).$$

Sketch a graph of *f(t,T) as a function of T, and derive expressions for p(t, T) and r(t,T).*

(15 marks)

QUESTION FIVE

a. Derive the Black-Scholes equation. (10 marks)

b. An investor buys, for a premium of 187.06, a call option on a non-dividend-paying stock whose current price is 5,000. The strike price of the call is 5,250 and the time to expiry is 6 months. The risk-free rate of return is 5% pa continuously compounded.

The Black-Scholes formula for the price of a call option on a non-dividend-paying share is assumed to hold.

(i) Calculate the price of a put option with the same time to maturity and strike price as the call. (5marks)

(ii) The investor buys a put option with strike price 4,750 with the same time to maturity. Calculate the price of the put option if the implied volatility were the same as that in (i).

[You need to estimate the implied volatility to within 1% pa of the correct value.]

(5 marks)