JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY

SCHOOL OF MATHEMATICS AND ACTUARIAL SCIENCE

UNIVERSITY EXAMINATION FOR DEGREE OF BACHELOR OF SCIENCE ACTUARIAL

4th YEAR 2nd SEMESTER 2021/2022

REGULAR (MAIN)

COURSE CODE: WAB 2406

COURSE TITLE: RISK AND CREDIBILITY THEORY.

EXAM VENUE: **STREAM: (BSc Actuarial Science)**

DATE: **EXAM SESSION:**

TIME: 2.00 HOURS

Instructions:

1. **Answer question 1 (Compulsory) and ANY other 2 questions**
2. **Candidates are advised not to write on the question paper.**
3. **Candidates must hand in their answer booklets to the invigilator while in the examination room.**

QUESTION ONE

a. State the credibility premium formula stating the essential features of risk premium calculated using credibility theory. (4marks)

b. A specialist insurer that provides insurance against breakdown of photocopying equipment calculates its premiums using a credibility formula. Based on the company's recent experience of all models of copiers, the premium for this year should be £100 per machine. The company's experience for a new model of copier, which is considered to be more reliable, indicates that the premium should be £60 per machine.

Given that the credibility factor is 0.75, calculate the premium that should be charged for insuring the new model. (2 marks)

c. Explain briefly what is meat y the “credibility factor”. (4marks)

d. Losses arising from a portfolio follow a Pareto distribution with parameters α=3 and λ=2,000.
Calculate the probability that a randomly chosen loss amount exceeds the mean loss amount. (4marks)

e. A loss amount random variable has MGF:
$M(t)=0.4(1-20t)^{-2} + 0.6(1-30t)^{-3}$
Calculate the expected loss amount. (3marks)

f. Claims arising from a certain type of insurance policy are believed to follow an exponential distribution. The lower quartile claim is 200.
Calculate the mean claim size. (4marks)

g. (i) State the two conditions that must hold for a risk to be insurable. (2marks)
(ii) List five other risk criteria that would be considered desirable by a general insurer.

(5marks)

h. An insurer is setting the premium rate for the buildings in an industrial estate. Past experience for the estate indicates that a premium rate of £3 per £1,000 sum insured should be charged. The past experience of other similar estates for which the insurer provides cover indicates a premium rate of £5 per £1,000 sum insured. If the insurer uses a credibility factor of 75% for this risk, calculate the premium rate per £1,000 sum insured. (2marks)

QUESTION TWO

a. A motor insurer wishes to estimate the claim frequency for a particular risk where claim numbers are assumed to have a Poisson distribution with parameter λ. λ is unknown but it is regarded as a random variable with a Gamma distribution with known parameters α and β.

(i)Given that the number of claims in the past n years are $X_1, X_2, ..., X_n$,
show that the Bayesian estimate of λ, with respect to quadratic loss function ,is

$$\frac{\alpha + \sum_{i=1}^{n} x_i}{\beta + n}$$

(10marks)

(ii)Explain how this result may be interpreted in terms of credibility theory, obtaining an expression for then credibility factor. (5marks)

b. The random variableS has a compound Poisson distribution with Poisson parameter 4. The individual claim amounts are either 1, with probability 0.3, or 3, with probability 0.7. Calculate the probability that S=4. (5marks)

QUESTION THREE

a. If X- Gamma (α,λ) , show that the MGF of X is:

$$M_X(t) = \left(1 - \frac{t}{\lambda}\right)^{-\alpha}$$

(5marks)

b. Based on an analysis of past claims, an insurance company believes that individual claims in a particular category for the coming year will be lognormally distributed with a mean size of £5,000 and a standard deviation of £7,500. Estimate the proportion of claims that will exceed £25,000. (10 marks)

c. Claims arising from a particular group of policies are believed to follow a Pareto distribution with parameters α and λ . A random sample of 20 claims gives values such that $\sum x=1,508$ and $\sum x^2 =257,212$. Estimate α and λ using the method of moments. (5marks)

QUESTION FOUR

Suppose that an insurance company uses an exponential distribution to model the cost of repairing insured vehicles that are involved in accidents, and the average cost of repairing a random sample of 1,000 vehicles is £2,200. A breakdown of the repair costs revealed the following numbers in different bands:

Repair cost, £	Observed number
0 – 1,000	200
1,000 – 2,000	300
2,000 – 3,000	250
3,000 – 4,000	150
4,000 – 5,000	100
5,000+	0

use this information to test whether the exponential distribution provides a good model for the individual repair costs. (5marks)

c. A compound random variable $S= X_1+ X_2 + \dots + X_N$ has claim number distribution: $P(N=n)= 9(n+1)4^{-n-2}$, n=0,1,2,…

The individual claim size random variable, X , is exponentially distributed with mean 2. Calculate E(S) and Var(s). (10marks)

d. The probability of a claim arising on any given policy in a portfolio of 1,000 one-year term assurance policies is 0.004. Claim amounts have a gamma(5,0.002) distribution. Calculate the mean and variance of the aggregate claim amount. (5marks)

QUESTION FIVE

a. A group of policies can give rise to at most two claims in a year. The probability function for the number of claims is as follows:

Number of claims, n	0	1	2
P(N=n)	0.6	0.3	0.1

Each claim is either for an amount of 1 or an amount of 2, with equal probability. Claim amounts are independent of one another and are independent of the number of claims.

Determine the distribution function of the aggregate annual claim amount, S. (5marks)

b. Determine an expression for the MGF of the aggregate claim amount random variable if the number of claims has a Bin(100,0.01) distribution and individual claim sizes have a gamma(10,0.2) distribution. (5marks)

c. The annual number of claims from a small group of policies has a Poisson distribution with a mean of 2. Individual claim amounts have the following distribution:

Amount	200	400
Probability	0.7	0.3

Individual claim amounts are independent of each other and are also independent of the number of claims. The insurer has purchased aggregate excess of loss reinsurance with a retention limit of 600.
Calculate the probability that the reinsurer is involved in paying the claims that arise in the next policy year. (10marks)