JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY

SCHOOL OF MATHEMATICS AND ACTUARIAL SCIENCE

UNIVERSITY EXAMINATION FOR DEGREE OF BACHELOR OF EDUCATION AND ACTUARIAL SCIENCE

4th YEAR 2ND SEMESTER 2022/2023 ACADEMIC YEAR

MAIN CAMPUS

COURSE CODE: WAB 2408

COURSE TITLE: RISK MATHEMATICS

EXAM VENUE: **STREAM: ACTUARIAL SCIENCE**

DATE: **EXAM SESSION:**

TIME: 2.00 HOURS

Instructions:

1. **Answer question one (compulsory) and any other two questions.**
2. **Candidates are advised not to write on the question paper.**
3. **Candidates must hand in their answer booklets to the invigilator while in the examination room.**

Question 1 [30 marks]

a) State four assumptions of Black Scholes Model [4marks]

b) Derive the formula for the MGF of the standard normal distribution [4marks]

c) Explain three assumptions of the consumer's preference 3marks

d) An investor buys a premium of 187.06 whose stock price is Ksh. 5000. The strike price is Ksh. 4750, and time to expiry is 6 months and has a volatility of 17%. Calculate the price of the put option if the interest rate is 5% [5marks]

e) Claims occur as a generalized Pareto distribution with parameter $\alpha = 6, \lambda = 200$ and $k = 4$. A proportional reinsurance arrangement is in force with a retained proportion of 80%. Find the mean and variance of the amount paid by the insurer and the reinsurer on an individual claim. [5marks]

f) The stock price 6 months from expiry of an option is £42, the exercise is £40, the risk-free interest rate is 10% p.a and the volatility is 20%p.a. calculate the price of a European [5marks]

g) Derive the expression for Var (S) in terms of the mean and Var (N) and Xi. [4marks]

Question 2 [20 marks]

Let N be the number of claims on a risk in one year. Suppose claims $x_1, x_2, \dots \dots x_N$ are independent, identically distributed random variables, independent of N . Let S be the total amount claimed in one year.

(i) Derive $E(S)$ and var(S) in terms of the mean and variance of N andX_i. [5marks]

(ii) Derive an expression for the moment generating function $M_S(t)$ of S in terms of the moment generating functions M_X (t) and $M_N(t)$ of X_I and N restively. [5marks]

(iii) If N has a Poisson distribution with mean λ show that:

$M_S(t) = \exp(\lambda\ (M_X\ (t)\text{-}1))$ [5marks]

(iv) If N has a binomial distribution with parameters m and q , determine the moment generating function of S in terms of m , q and M_X (t) . [5marks]

Question 3 [20 marks]

i) State 3 differences between individual risk Model and collective risk model. [6mrks]

ii) The table below shows the number of employees, probability of death, and the values of μ and σ for each age group. An insurer provides a one-year life cover. On death of an employee an insurer pays a death benefit to that employee. The insurer classifies life into 3 age groups and makes the following assumptions.

1. the probability of death is the same for people in same age-group
2. the death benefit of an employee is again in a given age group can be considered as a random variable N ~(μ, σ^2) distribution.
3. Lives are independent with respect to mortality.

Age-group	No. of employees	Probability of	μ	Σ

		death		
15-30	5000	0.0065	16000	5000
31-45	3000	0.0005	20000	4500
45-55	800	0.0065	25000	4800

Calculate the mean and the variance of the aggregate claims in a year. [14marks]

Question 4 [20 marks]

A statistician wishes to find a Bayesian estimate of the mean of an exponential distribution with density function $f(x) = \frac{1}{\mu} e^{-x/\mu}$. He is proposing to use a prior distribution of the form:

$$\text{Prior}(\mu) = \frac{\theta^{\alpha} e^{-\theta/\mu}}{\mu^{\alpha+1}\Gamma(\alpha)} \qquad \mu = 0$$

You are given that the mean of this distribution is $\frac{\theta}{\alpha-1}$

a) Write down the likelihood function for μ, based on a random sample of values $x_1 \dots \dots . . x_n$ from an exponential distribution. [2marks]

b) Find the form of the posterior distribution for μ, and hence show that an expression for the Bayesian estimate for μ under squared error loss is: [6marks]

$$\hat{\mu} = \frac{\theta + \sum x_i}{n - \alpha - 1}$$

c) Show that the Bayesian estimate for μ can be written in the form of a credibility estimate, and write down a formula for the credibility factor. [5marks]

d) The statistician now decides that he will use a prior distribution of this form with parameters $\theta = 40 \; and \; \alpha = 1.5$. His sample data have statistics $n = 100, \;\; \sum x_i = 9{,}826$ and $\sum x_i^2 = 1{,}200{,}000$. Find the posterior estimate for μ, and the value of the credibility factor in this case. [5marks]

e) Comment on the results obtained in part d [2marks]

Question 5 [20 marks]

a) The aggregate claims arising during each year from a particular type of annual insurance policy are assumed to follow a normal distribution with mean 0.7P and standard deviation 2.0 P where P is the annual premium. Claims are assumed to arise independently. Insurers assess their solvency position at the end of each year. A small insurer with an initial surplus of £0.1m expects to sell 200 policies during the second year for an annual premium of £5,000 and expects to incur expenses of 0.2P, calculate the probability that the insurer will prove to be insolvent at the end of the second year. [10marks]

b) Suppose the annual frequency of losses from a portfolio follows a Poisson distribution with parameter $\lambda = 10$ such that $P(N = k) = \frac{e^{-10}10^k}{K!}$ for k=0,1,2… Suppose the individual amounts X are uniformly distributed on the interval [0, 1000]. Compute the mean and the variance of the aggregate losses against the portfolio. [10marks]