JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY

SCHOOL OF MATHEMATICS AND ACTUARIAL SCIENCE

UNIVERSITY EXAMINATION FOR DEGREE OF BACHELOR OF SCIENCE ACTUARIAL

3RD YEAR ST SEMESTER 2022/2023

REGULAR (MAIN)

COURSE CODE: WAB 2303

COURSE TITLE: ACTUARIAL MATHEMATICS II

EXAM VENUE: **STREAM: (BSc Actuarial Science)**

DATE: 19/12/2022 EXAM SESSION: 9.00-11.00AM

TIME: 2.00 HOURS

Instructions:

1. **Answer question 1 (Compulsory) and ANY other 2 questions**
2. **Candidates are advised not to write on the question paper.**
3. **Candidates must hand in their answer booklets to the invigilator while in the examination room.**

QUESTION 1

Define the following terms

i.	Pension	(2MARKS)
ii.	Premiums	(2MARKS)
iii.	Profit vector	(2MARKS)
iv.	Profit signature	(2MARKS)
v.	Reversionary annuities	(2MARKS)

a. Determine which of the following assertions relating to a multiple decrement table are correct:

   i. The dependent probabilities of decrement can never exceed the corresponding independent probabilities. (1MARKS)

   ii. Forces of decrement can never exceed 1 in value. (1MARKS)

   iii. The total of all the decrement numbers summed over all ages equals the initial radix of the table (1MARKS)

b. You are given the following extract from a double decrement table:

Age x	$(al)_x$	$(ad)_x^w$	$(ad)_x^w$
55	100,000	175	2,490
56	97,335 180	2,160	
57	94,995		

Where *d* and *w*refer to death and withdrawal respectively.

Calculate:

i.	$(ap)_{51}$	(2MARKS)
ii.	$(aq)_{51}^w$	(2MARKS)
iii.	$2(aq)_{51}^w$	(2MARKS)
iv.	$_2(ap)_{50}$	(2MARKS)
v.	$_{1|}(aq)_{50}^d$andstatewhat thisprobabilitymeansin words.	(3MARKS)

c. calculate the followings ,

i.	$_5q_{40}{}^1{}_{:40}$	(3 MARKS)
ii.	$_5q_{40}{}^2{}_{:40}$	(3 MARKS)

Using AM92 Mortality

QUESTION 2

a. Active members of a pension scheme are subject to the following probabilities of decrement at the given ages (where *r* and *d* stand for retirement and death, respectively).

Age x	$(aq)_x^r$	$(aq)_x^d$
60	0.01	0.03
61	0.02	0.04

Calculate the following probabilities, all relating to an active member who is currently exactlyaged 60.

i. The probability of retiring during the year of age 61 to 62. (2MARKS)

ii. The probability of dying as an active member before age 62. (2MARKS)

iii. The probability of still being an active member at age 62. (2MARKS)

b. A population of healthy people over the year of age 50 to 51 is subject to a constant force of Decrement due to sickness of 0.08 per annum, and a constant force of mortality of 0.002 per annum. Assuming that a double decrement model is used, calculate:

i. the probability that a healthy person aged exactly 50 will still be healthy at exact age 51 (2MARKS)

ii. the probability that a healthy person aged exactly 50 will leave the healthy population through death before exact age 51 (3MARKS)

iii. The independent probability of a life aged exactly 50 dying before exact age 51. (3MARKS)

c. Calculate $a_{65|60}$, assuming:

(i) (65) is male and (60) is female (3MARKS)

(ii) (65) Is female and (60) is male. (3MARKS)

Basis: Mortality: PMA92C20 for the male life, PFA92C20 for the female life

Interest: 4% *pa* effective

QUESTION 3

a. In a certain population, forces of decrement are assumed to be constant over individual years of age. The following independent forces of decrement will be assumed for this population between the exact ages of 50 and 52: Force of decrement for year of age commencing from exact age *x*

Age *x*	due to mortality	due to sickness
50	0.016	0.075
51	0.012	0.089
52	0.070	0.045

Construct a double decrement table including the two decrements of mortality and sickness, forthis population between exact ages 50 and 53, assuming a radix of $(al)_{50}$=1000,000 .

(10MARKS)

b. Two lives are aged 50 and 60. The 50-year-old is subject to a constant force of mortality of 0.02 *pa*, and the 60-year-old is subject to a constant force of mortality of 0.025 *pa*. Assuming that the constant force of interest is 6% *pa*, calculate:

   i. $A_{50^1:60}$ (4MARKS)

   ii. $A_{50:60^1}$ (4MARKS)

c. Define contingent assurances (2MARKS)

QUESTION 4

a. An endowment assurance pays 100,000 in three years' time or immediately on the earlier death ofa life aged 50 at entry. On surrender at any time, a surrender value equal to 70% of the premiumspaid by the date of surrender (without interest) is payable. Surrender payments are assumed tooccur immediately at the time of surrender. A level annual premium of 9,000 is paid at the startof each year.Calculate the expected present value of the death, maturity and surrender benefits for a singlepolicy at outset, using the following assumptions:

Mortality: AM92 Ultimate

Annual force of decrement due to surrender: 0.3 in year 1,

and 0.07 in each of years 2and 3

Interest: 3% per annum compound

State any further assumptions you make (20MARKS)

QUESTION 5

A life insurance company sells a 3-year regular-premium endowment assurance policy to a 55-year-old male. The sum insured is £10,000 (payable at the end of year of death). Initial expenses are 50% of annual premium, renewal expenses are 5% of subsequent premiums. Premiums are payable annually in advance. There is a surrender benefit payable equal to return of premiums paid, with no interest. This is paid at the end of the year of withdrawal.

The company is required to hold net premium reserves, calculated ignoring surrenders. Calculate the projected yearly cashflows per policy in force at the start of each year, using the following bases.

For pricing: AM92 Ultimate mortality, 4% pa interest, expenses as above and ignoring surrenders.

For valuation: Interest and mortality as per pricing.

For future cashflow projection: Interest and expenses as per pricing, dependent surrender and mortality probabilities as in the table below.

Age x	$(aq)_x^w$		$(aq)_x^w$
55	0.005	0.1	
56	0.006		0.05
57	0.007	0.05	
58	0.008		0.01
59	0.009	0	

(20MARKS)